LECTURE NOTES ON FLUID MECHANICS

MECHANICAL BRANCH

4TH SEM

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1·0 PROPERTIES OF FLUID :-

1·1 - Define Fluid :-

→ Fluid may be defined as a substance which is capable of flowing. It has no definite shape of its own, but it takes the shape of the containing vessel.

→ Further even a small amount of shear force exerted on a fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.

→ The fluids are also classified as ideal fluid and real fluid. Ideal fluids are those fluids which have no viscosity and surface tension and they are incompressible. Ideal fluids are only imaginary fluids.

→ Real fluids are those fluids which are actually available in nature. These fluids possess the properties such as viscosity, surface tension and compressibility.

1·2 properties :-

(a) Density (ρ) :-

→ It is defined as the ratio of mass and volume.

→ It is denoted by symbol (ρ).

→ SI unit of density is (Kg/m^3).

→ $\boxed{\rho = m/V}$

→ density of water $(\rho)_w = 1000\ Kg/m^3$.

→ density of air $\rho_{air} = 1.208\ Kg/m^3$.

→ density of seawater $(\rho)_{s.w} = 1025\ Kg/m^3$.

(b) Specific Weight :-

→ It is defined by weight per unit volume.

→ It is denoted by symbol 'w'.

→ SI unit of specific weight is (N/m^3).

→ $\boxed{w = \frac{weight}{volume}}$

→ $\boxed{w = \frac{mg}{V} = \left(\frac{m}{V}\right) \times g = \rho \times g}$

→ specific weight of water = 1000 × 9.81 = 9810 N/m³

→ 'ω' depends upon g and density. so its value also depends upon temperature and pressure.

<c> Specific volume :-

→ specific volume is generally defined as the volume of the fluid per unit mass.

→ It is reciprocal of density.

→ In SI unit the specific weight is expressed in (m³/kg).

→ It is denoted by 'υ'.

$$\boxed{\upsilon = 1/\rho}$$

<d> Specific Gravity :-

→ specific gravity is defined as the ratio of density of fluid to the density of standard fluid.

→ For liquids, standard fluid is taken as water and for gases the standard fluid is taken as air.

→ It is denoted by 's'.

$$\boxed{S = \frac{\text{Density of fluid}}{\text{Density of standard fluid}}}$$

→ The value of specific gravity of water = 1

* Problem-1

calculate the specific weight, density and specific gravity of 1 lt of a liquid which weighs 7N.

Data given:-

volume = 1 litre = $\left(\frac{1}{1000}\right)$ m³

weight = 7 N.

(i) specific weight (ω) = $\frac{\text{weight}}{\text{volume}} = \frac{7\ N}{\left(\frac{1}{1000}\right) m^3} = 7000\ N/m^3$

(ii) Density (ρ) = $\frac{\text{specific weight}}{g} = \frac{7000}{9.81} = 713.5\ kg/m^3$

(iii) Specific gravity = $\frac{\text{density of liquid}}{\text{density of water}}$

$= \frac{713.5}{1000} = 0.7135$

1.3 viscosity:-

→ viscosity is defined as the property of fluid which offers the resistance to the movement of one layer of fluid over another adjucent layer of the fluid.

→ when two layers of a fluid at distance 'dy' apart moves one over the other at different velocities say 'u' and 'u+du' the viscosity together with relative velocity causes a shear stress acting between the fluid layers.

→ This shear stress is proportional to the change of velocity with respect to 'y'.

$$\tau \propto \frac{du}{dy}$$

$$\Rightarrow \tau = \mu\left(\frac{du}{dy}\right)$$

→ where μ is the constant of proportionality and is known as coefficient of dynamic viscosity.

$$\mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

unit of dynamic viscosity:-

$$\mu = \frac{\text{shear stress}}{\left(\frac{\text{change in velocity}}{\text{change in distance}}\right)} = \frac{\text{Force/area}}{\text{length/time} \times \frac{1}{\text{length}}}$$

$$= \frac{N/m^2}{1/s} = \frac{Ns}{m^2}$$

→ SI unit of μ = Ns/m^2 = pa·sec

→ SI unit of viscosity is called as poise.

→ In CGS the unit of viscosity

$$1 \text{ poise} = \frac{\text{Dyne s}}{cm^2}$$

$$1 \text{ poise} = \frac{1}{10}\frac{Ns}{m^2}$$

→ sometimes it is expressed in centipoise.

$$1 \text{ centipoise} = \frac{1}{100} \text{ poise}$$

→ Kinematic viscocity :-

It is defined as the ratio of dynamic viscocity and density of fluid. It is denoted by (ν).

→ $$\boxed{\nu = \frac{\text{Dynamic Viscocity}}{\text{Density}} = \frac{\mu}{\rho}}$$

→ unit of kinematic viscocity can be obtained by

$$\nu = \frac{\text{unit of } \mu}{\text{unit of } \rho} = \frac{\text{Force} \times \text{time}}{(\text{length})^2 \times \frac{\text{mass}}{(\text{length})^3}} =$$

$$\frac{\text{Force} \times \text{time}}{\left(\frac{\text{mass}}{\text{length}}\right)} = \frac{N/m^2 \times sec}{(Kg/m)}$$

$$= \frac{Kg \times m/sec^2 \times sec}{(Kg/m)}$$

$$\boxed{\nu = m^2/sec}$$

→ SI unit of kinematic viscocity is (m^2/sec).

→ CGS unit of kinematic viscocity is (cm^2/sec).

→ In CGS, kinematic viscocity is written as stoke.

$$1 \text{ stoke} = 1\, cm^2/sec = \left(\frac{1}{100}\right)^2 m^2/sec = 10^{-4}\, m^2/sec.$$

* Newton's Law of Viscocity :-

→ It states that the shear stress (τ) on a fluid element is directly proportional to rate of shear strain.

$$\tau \propto \left(\frac{du}{dy}\right) \Rightarrow \boxed{\tau = \mu\left(\frac{du}{dy}\right)}$$

→ The fluids which obey the above relation is called newtonian fluid. and the fluids which don't obey the above relation are called non-newtonian fluid.

problem

Two horizontal plates are kept 1.2 cm apart. The space between them being filled with oil of viscosity 14 poise. Calculate the shear stress in oil if upper plate is moving with a velocity of 2.5 m/sec.

$dy = 1.25\text{ cm} = 0.0125\text{ m}.$

$\mu = 14 \text{ poise} = \left(\frac{14}{10}\right) NS/m^2$

shear stress $\tau = \mu\left(\frac{du}{dy}\right)$

$\Rightarrow \tau = \frac{14}{10} \times \frac{2.5 - 0}{0.0125}$

$$\boxed{\tau = 280 N/m^2}$$

* SURFACE TENSION :-

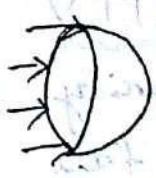
→ Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immisible liquids such that the contact surface behaves like a membrane under tension.

→ The magnitude of the force per unit length of the free surface will have the same value as the surface energy per unit area.

→ It is denoted by (σ).

→ SI unit of surface tension (N/m).

* Surface tension on liquid droplet :-



→ Consider a small spherical droplet of a liquid of radius 'r', on the entire surface the tensile force is acting due to surface tension.

→ If the droplet is cut into two halves, the force acting on one half will be

(i) The tensile force due to surface tension acting around the circumference of the cut portion

$= \sigma \times \pi d$

→ force on the area $= P \times \frac{\pi}{4} d^2$

$\Rightarrow P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$

$\Rightarrow \boxed{P = \frac{4\sigma}{d}}$

* **Surface tension on soap bubble :-**

A hollow bubble like a soap bubble in air has two surfaces in contact with air one inside and one outside.

→ Thus two surfaces are subjected to surface tension

$\Rightarrow P \times \frac{\pi}{4} d^2 = 2 \times \sigma \times \pi d$

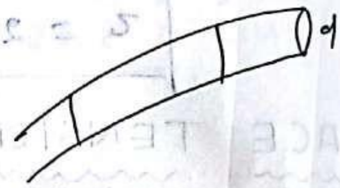
$$\Rightarrow \boxed{P = \frac{8\sigma}{d}}$$

* **Surface tension on liquid jet :-**

$\Rightarrow P \times A = \sigma \times (2l)$

$\Rightarrow P \times l \times d = \sigma \times 2l$

$$\Rightarrow \boxed{P = \frac{2\sigma}{d}}$$



problem :-

Find the surface tension in a soap bubble of 40mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Given $d = 40mm = 40 \times 10^{-3}$ m

$P = 2.5 \, N/m^2$

$\Rightarrow P = \frac{8\sigma}{d}$

$\Rightarrow 2.5 = \frac{8\sigma}{40 \times 10^{-3}}$

$$\Rightarrow \boxed{\sigma = 0.0125 \, N/m}$$

CAPILLARITY :-

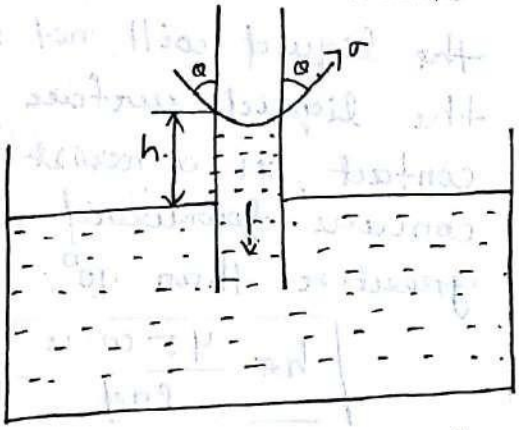
→ capillarity is defined as a phenomenon of rise or fall of a liquid when the tube is held vertically in the liquid.

→ The rise of liquid surface is known as capillary rise whereas the fall of liquid surface is known as capillary fall.

→ It is expressed in cm of liquid.

→ It's value depends upon the specific weight of liquid, diameter of tube and surface tension of the liquid.

* Capillary rise



→ If molecules of certain liquid possess greater affinity for solid molecules or liquid has greater adhesion than cohesion, then it will wet the solid surface with which it is in contact and tend to rise at the point of contact.

→ The liquid surface will be concave upward and the angle of contact is < 90°. It is known as capillary rise.

→ h = height of liquid in tube

σ = Surface tension of liquid.

θ = angle of contact b/w liquid and glass tube.

weight = F_σ (vertical)

$\gamma \times \pi/4\, d^2 \times h = \sigma \cos\theta \times L$

$\Rightarrow \gamma \times \pi/4\, d^2 \times h = \sigma \cos\theta \times (\pi d)$

$\Rightarrow \gamma \times \pi/4\, d^2 \times h$

$$\Rightarrow h = \frac{\sigma \cos\theta \times (\pi d)}{\pi/4\, d^2 \times \rho g}$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

If $\theta = 0°$ then $\cos\theta = 1$

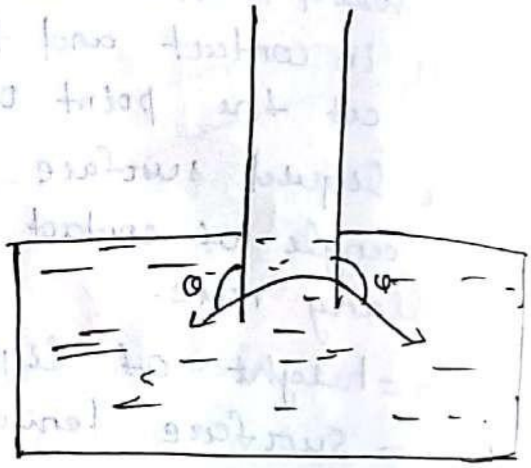
$$h = \frac{4\sigma}{\rho g d}$$

* Capillary Fall

on the other hand, If for any liquid there is less attraction for solid molecules or in the other words cohesion predominates, the liquid the liquid will not wet the solid surface and the liquid surface will fall at the point of contact, as a result the liquid surface will be concave downward and the angle of contact (θ) is greater than 90°.

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

→ value of θ for mercury and glass tube is 128°.



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Q) calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tension $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is given 13.6 and the angle of contact = 130°.

$d = 2.5\,mm = 2.5 \times 10^{-3}\,m$

$(\sigma)_{w} = 0.0725\,N/m$

$(\sigma)_{Hg} = 0.52\,N/m$

$(S)_{Hg} = 13.6$

$\rho = 13.6 \times 10^3\,kg/m^3$

capillary rise for water

$$h = \frac{4\sigma}{\rho g d} = \frac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= 0.0118 \text{ m} = 1.18 \text{ cm}.$$

capillary rise for mercury

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

$\theta = 130^\circ$

$$h = \frac{4 \times 0.52 \times \cos 130^\circ}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$$

$$= -0.4 \text{ cm}$$

*. negative sign indicates the capillary depression.

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2.0 :- FLUID PRESSURE AND ITS MEASUREMENT :-

pressure intensity :-

pressure intensity may be defined as the force exerted on a unit area. If 'F' represents total force uniformly distributed over an area 'A', the pressure at any point $p = (F/A)$. If the force is not uniformly distributed, the expression will give the average value only.

→ When the pressure varies from point to point on an area, the magnitude of pressure at any point can be obtained

$$p = \frac{dF}{dA}$$

unit :-

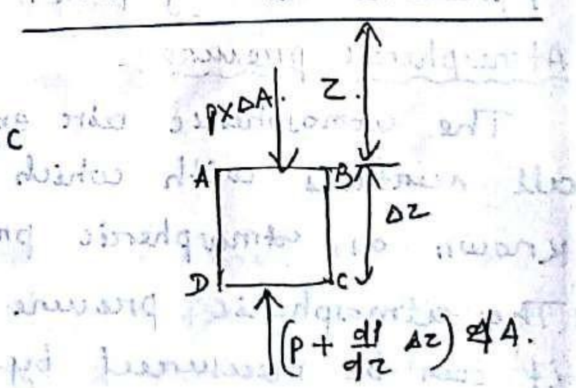
SI unit of pressure N/m^2 or pascal.

$$1\ Kpa = 1000\ pa = 10^3\ N/m^2$$

$$1\ bar = 10^5\ pa = 10^2\ Kpa = 100\ Kpa = 10^5\ N/m^2.$$

pressure variation in a fluid at rest :-

→ The pressure at any point in a fluid at rest is obtained by the hydrostatic law, which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.



→ ΔA = cross sectional area of the element.

ΔH = height of the fluid element.

p = pressure on face AB

Z = distance of fluid element from free surface.

→ pressure force on AB $= p \times \Delta A$

$$CD = \left(p + \left(\frac{dp}{dz}\right)\Delta z\right) \times \Delta A$$

→ weight of fluid = $\rho \times g \times (\Delta A \times \Delta Z)$

pressure force on AB and CD are equal and opposite

$$\frac{\partial p}{\partial z} = \rho \times g = w$$

$$\int dp = \int \rho g dz$$

$$\Rightarrow p = \rho g z$$

→ where p is the pressure above atmospheric pressure and Z is the height of the point from free surface.

$$Z = \frac{p}{\rho g}$$

→ Z is called pressure head.

2.2 Pascal's Law :-

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all direction.

$$P_x = P_y = P_z$$

→ pressure at any point in x, y, z directions is equal.

2.3 Atmospheric pressure :-

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact. It is known as atmospheric pressure.

→ The atmospheric pressure varies with altitude and it can be measured by using barometer.

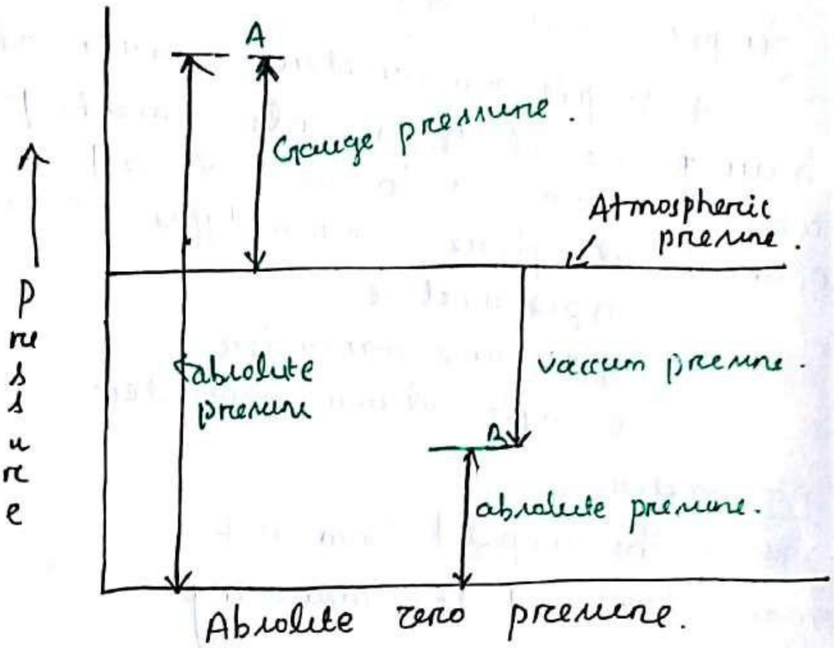
→ value of atmospheric pressure = 101.325 kPa. or 10.3 m of water or 76 cm. of mercury.

Absolute pressure :-

The pressure measured with reference to absolute vacuum/zero (complete vacuum) then that is called as absolute pressure.

Gauge Pressure :-

→ It is defined as the pressure which is measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum.



vaccum pressure :-

→ It is defined as the pressure below the atmospheric pressure.

* Absolute pressure = Atmospheric pressure + gauge pressure

$$P_{ab} = P_{atm} + P_g$$

* Absolute pressure = Atmospheric pressure - vaccum pressure.

$$P_{ab} = P_{atm} - P_v$$

2.4 pressure Measuring Instruments :-

pressure is measured by following devices

① Manometers.

② Mechanical gauge.

Manometers :- Manometers are defined as devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as

(1) simple Manometers.

(2) Differential Manometers.

Mechanical Gauge :- Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :-

(1) Diaphragm pressure gauge

(2) Bourdon tube pressure gauge

(3) Dead-weight pressure gauge.

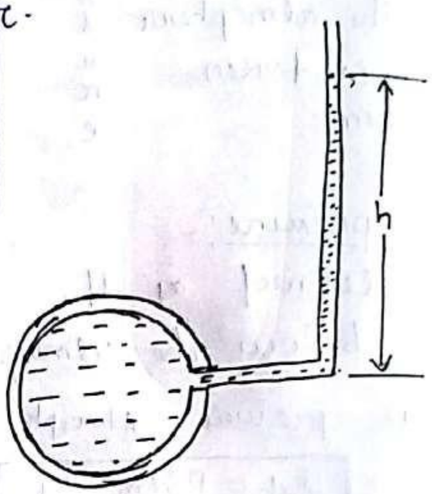
(4) Bellows pressure gauge.

Simple Manometers :-

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are

① piezometer
② U-tube manometer
③ single column manometer.

piezometer :-



→ It is the simplest form of manometer used for measuring gauge pressure.

→ one end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere.

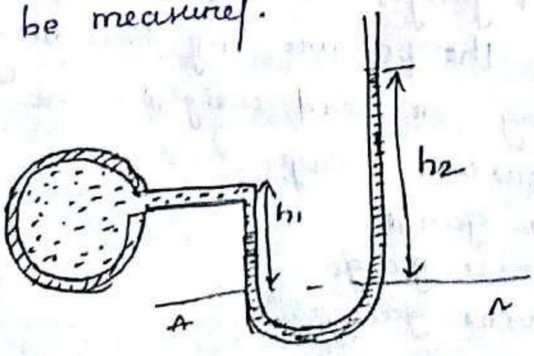
→ The rise of liquid gives the pressure head at that point.

→ The height of liquid is 'h' in piezometer tube the pressure at 'A' = $\rho g \times h$ N/m^2

U-tube Manometer :-

→ It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere.

→ The tube generally ~~consists of~~ contains mercury / or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.



for gauge pressure.

for gauge pressure :-

Let B is the point at which pressure is to be measured, whose value is P. The datum line is A-A.

h_1 = Height of liquid above datum line.
h_2 = height of heavy liquid above datum line.
S_1 = specific gravity of light liquid
S_2 = specific gravity of heavy liquid
ρ_1 = Density of light liquid $= 1000 \times S_1$
ρ_2 = density of heavy liquid $= 1000 \times S_2$

As the pressure is the same for horizontal surface, so the pressure above the horizontal datum line A-A in the left column, and in the right column of U-tube manometer should be same.

pressure above A-A in the left column $= p + \rho_1 g h_1$

pressure above A-A in the right column $= \rho_2 g h_2$.

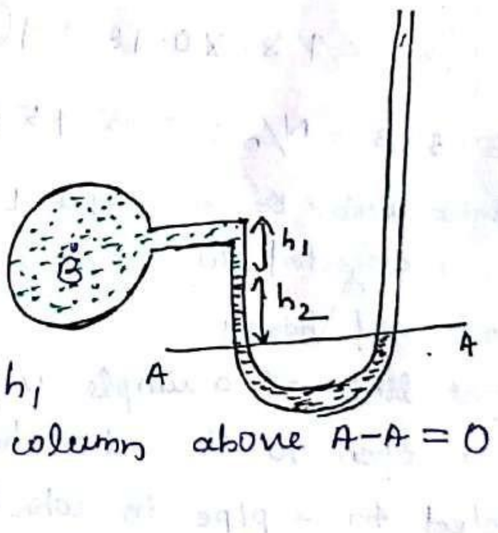
Hence equating the two pressure

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$\Rightarrow \boxed{p = (\rho_2 g h_2 - \rho_1 g h_1)}$$

for vaccum pressure :-

for measuring vaccum pressure the level of the heavy liquid in the manometer will be.



the pressure above A-A in the left column $= p + \rho_2 g h_2 + \rho_1 g h_1$

pressure in the right column above A-A = 0.

pressure head

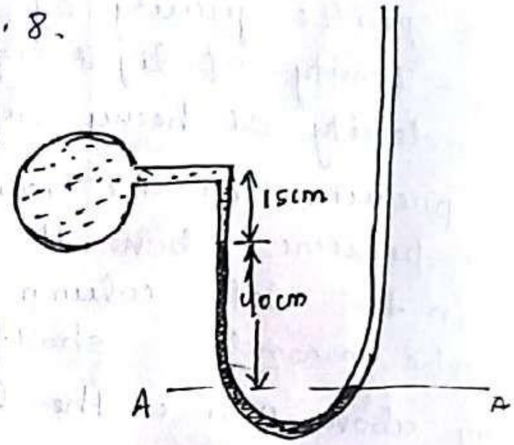
$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\Rightarrow \boxed{p = -(\rho_2 g h_2 + \rho_1 g h_1)}$$

(Q)(1) Simple U-tube manometer containing mercury is connected to a pipe in which fluid of sp. gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vaccum pressure in the pipe if the difference of mercury level in the two limbs is 40cm and the height of fluid in the left from the centre of pipe is 15cm. below.

Ans Specific gravity of liquid $S_1 = 0.8$.

sp. gravity of $Hg = 13.6$.



ρ. of liquid $(\rho_1) = S_1 \times 1000$
$= 0.8 \times 1000$
$= 800$.

ρ of mercury $(\rho_2) = 13.6 \times 1000$
$= 13600$.

$h_1 = 15cm = 0.15m$.

$h_2 = 40cm = 0.4m$.

$p + \rho_2 g h_2 + \rho_1 g h_1 = 0$.

$\Rightarrow p = -(\rho_1 g h_1 + \rho_2 g h_2)$

$= -[(800 \times 9.81 \times 0.15) + (13600 \times 9.81 \times 0.4)]$

$= -54543.6 \, N/m^2 = -5.45 \, N/cm^2$ (Ans).

~~(Q)(2) The right limb of a simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gravity 0.8 and having~~

(Q) 2. The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gravity is 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury in the two limbs is 20cm.

SINGLE COLUMN MANOMETER :-

Single column manometer is a modified form a U-tube manometer in which a reservoir, having a large cross sectional area as compared to the area of the tube is connected to one of the limbs of the manometer. There are two types of single column manometer as :-

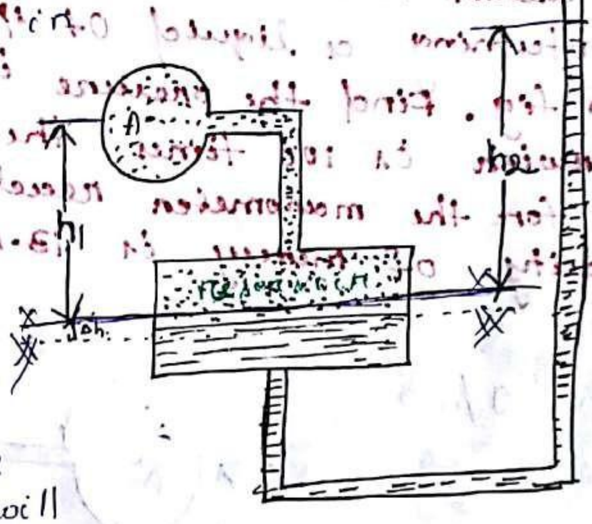
① Vertical single column Manometer

② inclined single column manometer.

① Vertical single column manometer :-

→ It shows the vertical single column manometer.

→ X-X be the datum line in the reservoir and in the right limb of the manometer.



→ When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

→ Δh = fall of heavy liquid in reservoir.

h_2 = rise of heavy liquid in right limb.

h_1 = height of centre of pipe above X-X which is to be measured.

p_A = pressure at A

A = cross sectional area of the reservoir.

a = cross sectional area of right limb

S_1 = specific gravity of liquid in pipe.

S_2 = sp. gravity of heavy liquid in reservoir

ρ_1 = density of liquid in pipe

ρ_2 = density of liquid in reservoir.

$$A \times \Delta h = a \times h_2$$

$$\Rightarrow \boxed{\Delta h = \left(\frac{a}{A}\right) \times h_2}$$

Pressure in the right limb above Y-Y

$$= \rho_2 \times g\,(\Delta h + h_2).$$

pressure in the left limb above Y–Y =

$\rho_1 \times g \times (\Delta h + h_1) + P_A$.

equating these pressures,

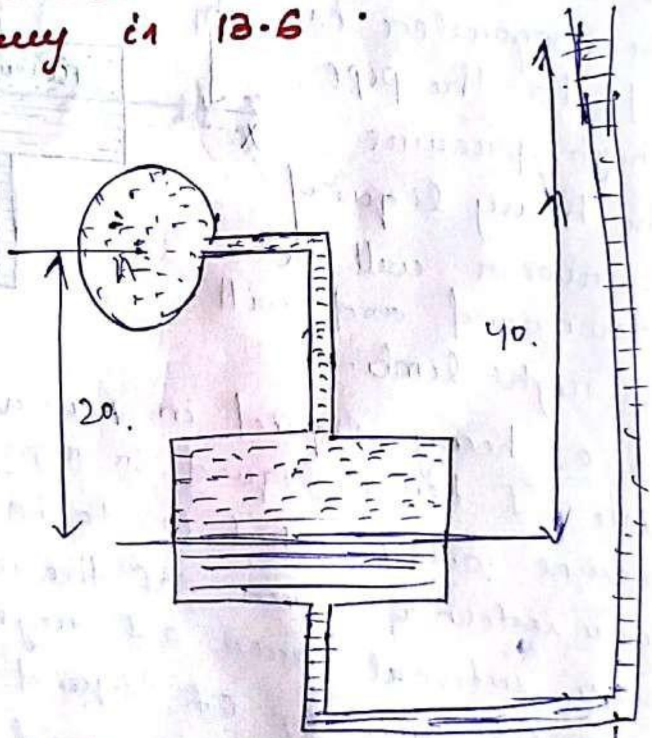
$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$

$\Rightarrow P_A = \rho_2 \times g\,(\Delta h + h_2) - \rho_1 \times g \times (\Delta h + h_1)$

$= \Delta h\,(\rho_2 g - \rho_1 g) + h_2 \rho_2 g - h_1 \rho_1 g$.

$$\boxed{\Delta h = \frac{a \times h_2}{A}}$$

Q) A single column manometer is connected to a pipe containing a liquid of sp. gravity 0.9 as shown in fig. Find the pressure in the pipe of the reservoir is 100 times the area of the tube for the manometer reading. The specific gravity of mercury is 13.6.



$S_1 = 0.9$

$\rho_1 = 900\ kg/m^3$

$\rho_2 = 13600$,,

b. $A/a = 100$.

$h_1 = 20cm = 0.2m$

$h_2 = 40cm = 0.4m$

$P_A = \frac{a}{A} h_2 [\rho_2 g - \rho_1 g] +$

$h_2 \rho_2 g - h_1 \rho_1 g$

$= 5.21\ N/cm^2$.

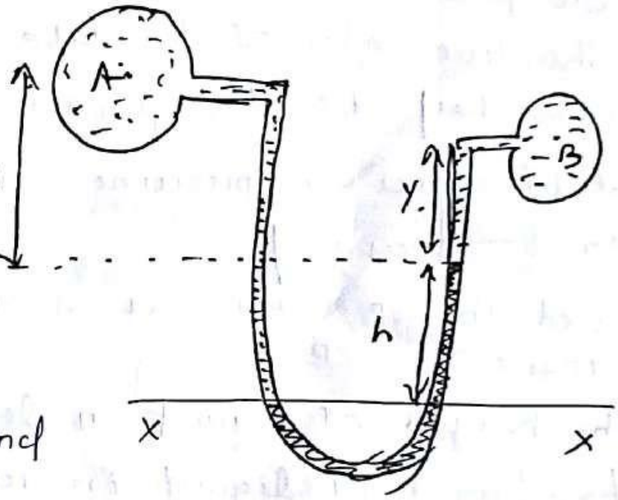
DIFFERENTIAL MANOMETERS

differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured.

(1) U-tube differential manometer

(2) Inverted U-tube differential manometer.

(1) U-tube differential Manometer :-



→ The two points A and B are at different level and also contains liquids at different sp. gravity. Three points are connected to the U-tube differential manometer.

→ Let the pressure at A and B are p_A and P_B.

→ h = difference of Hg level in the U-tube.

- y = difference of centre of B from Hg level in right limb.

x = difference of centre of A from Hg level in centre of 'A'.

ρ_1 = density of liquid at A

ρ_2 = " " " B

ρ_g = " " Hg.

pressure above x-x in left limb $= \rho_1 g(h+x) + P_A$

" " " " Right " $= \rho_g \times g \times h + \rho_2 \times g \times y + P_B$

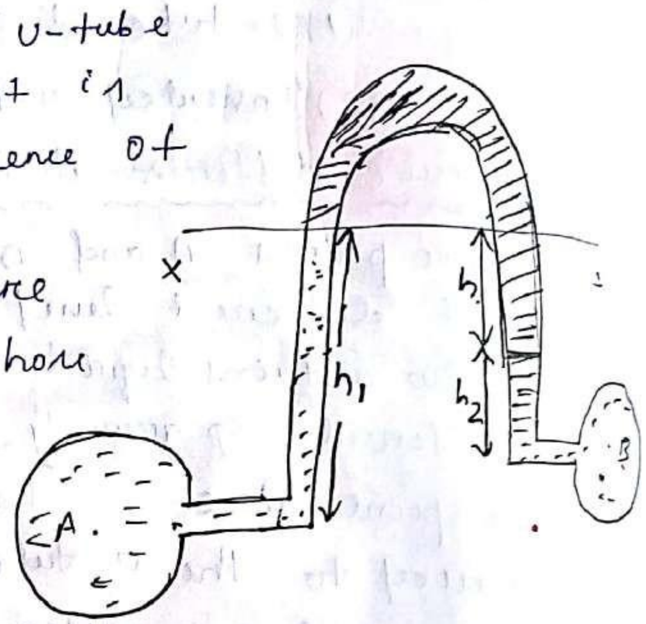
equating the two pressures

$$\rho_1 g(h+x) + P_A = \rho_g \times g \times h + \rho_2 \times g \times y + P_B$$

$$\Rightarrow P_A - P_B = \rho_g \times g \times h + \rho_2 \times g \times y - \rho_1 \times g\,(h+x)$$

$$= h \times g\,(\rho_g - \rho_1) + \rho_2 \times g \times y - \rho_1 \times g \times x.$$

Inverted U-tube differential manometer :-



→ It consists of an inverted U-tube containing a light liquid. It is used for measuring difference of low pressure.

→ The two ends of the tube are connected at two points whose difference in pressure is to be measured.

→ Let the pressure at A is more than B.

h_1 = height of liquid in left limb below X-X.

h_2 = height of liquid in right limb.

h = difference in light liquid.

ρ_1 = Density of liquid at 'A'.

ρ_2 = Density of liquid at 'B'.

ρ_g = density of light liquid.

P_A = pressure at 'A'

P_B = pressure at 'B'.

pressure in the left limb above X-X $= P_A - \rho_1 g \times h_1$

pressure in the right limb below X-X $= P_B - \rho_2 g h_2 - \rho_g g h$.

equating

$$P_A - \rho_1 g h_1 = P_B - \rho_2 g h_2 - \rho_g g h$$

$$\boxed{\Rightarrow P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_g g h}$$

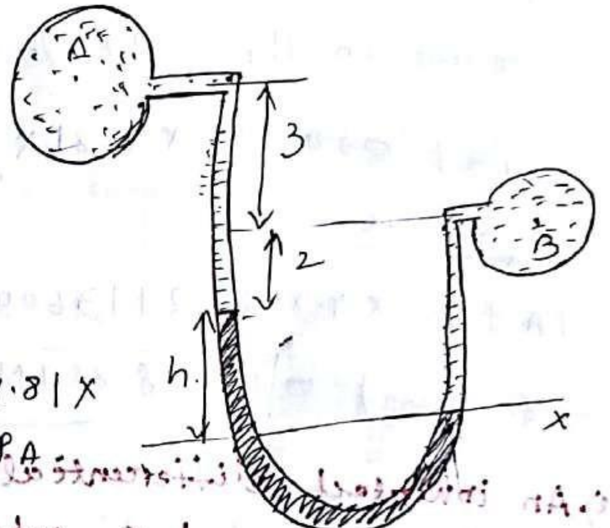
Q) A differential manometer is connected at the two points A and B of two pipes as shown. The pipe A contains a liquid of specific gravity = 1.5 while pipe B contains a liquid of sp. gravity = 0.9. The pressures at A and B are 1 Kgf/cm2 and 1.80 Kgf/cm2 respectively. find the difference in mercury level in differential manometer.

$P_A = 1 kgf/cm^2 = 1\times10^4 kgf/m^2$

$= 10^4 \times 9.81 N/m^2$

$P_B = 1.8 Kgf/cm^2$

$= 1.8 \times 9.81 \times 10^4 N/m^2$



Left limb

$= 13.6\times1000\times 9.81\times h + 1500\times9.81\times(2+3) + P_A$

Right limb

$= 900\times9.81\times(h+2) + P_B$

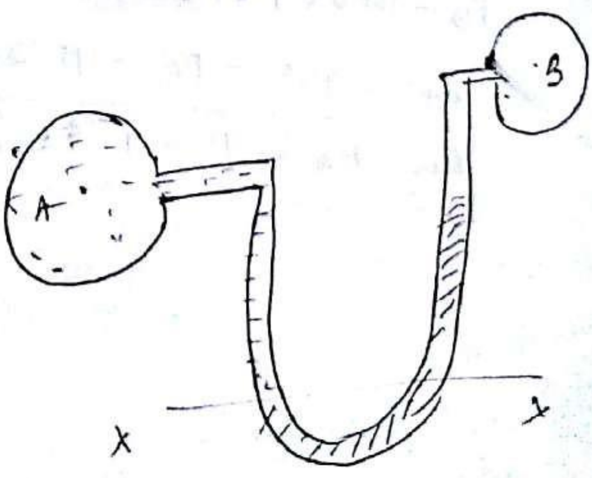
$h = 18.1 cm$

Q) A pipe contains an oil of sp. gravity of 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. find the difference of pressure at the two points.

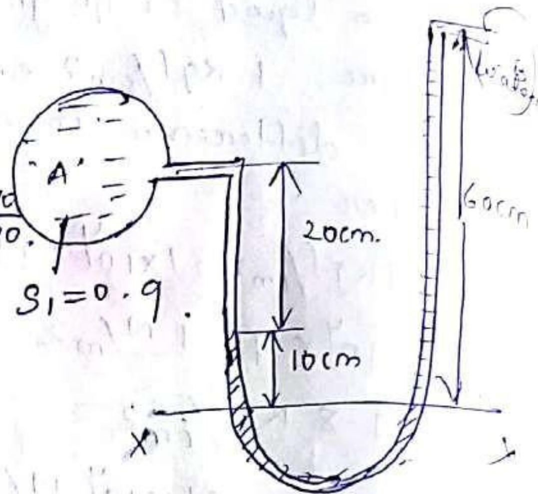
$S_1 = 0.9$ $\rho_1 = 0.9\times1000$

$h = 15 cm.$

$S_g = 13.6.$



1) A differential manometer is connected at the two points A and B as shown in figure. At B air pressure is 9.81 N/cm2. Find the absolute pressure at 'A'.



$\rho_1 = 0.9 \times 1000 = 900\ kg/m^3$

pressure in the left limb =

$P_A + 900 \times 9.81 \times \left(\frac{20}{100}\right) + 13600 \times 9.81 \times \frac{10}{100}$.

pressure in the right limb

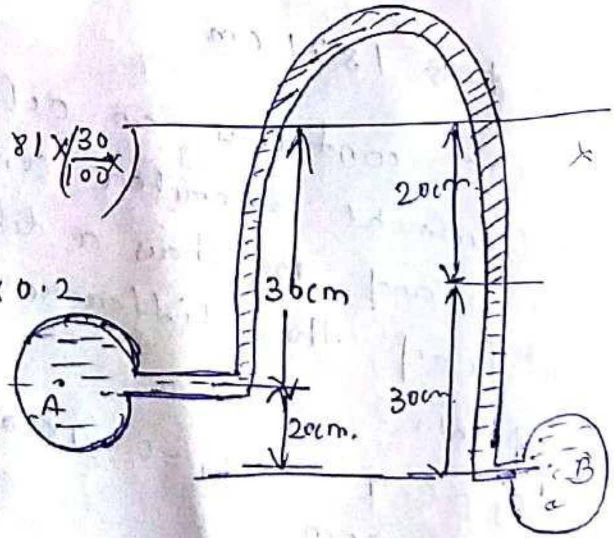
$= P_B + 1000 \times 9.81 \times \frac{60}{100}$.

equating

$P_A + 900 \times 9.81 \times 0.2 + 13600 \times 9.81 \times 0.1 = P_B + 9.81 \times 1000 \times 0.6$

$\Rightarrow$ $\boxed{P_A = 8.887\ N/cm^2}$

2) An inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gravity 0.8. Find the pressure difference b/w A and B.



$S_1 = 0.8$.

$\rho_1 = 800\ kg/m^3$.

in the left limb $= P_A - 1000 \times 9.81 \times \left(\frac{30}{100}\right)$

in right limb =

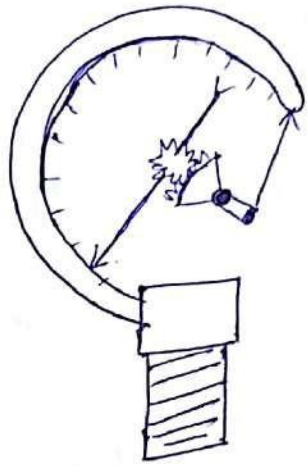
$P_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$\Rightarrow P_A - 2943 = P_B - 4512.6$

$\Rightarrow \boxed{P_B - P_A = 1569.6\ N/m^2}$

Ans

Bourdon Tube Pressure Gauge:-



→ It is the most common type of pressure gauge which was invented by E-Bourdon.

→ The pressure responsive element in this gauge is a tube of steel or bronze which is of elliptical cross-section and curved into a circular arc.

→ The tube is closed at its outer end, and this end is free to move.

→ The other end of the tube through which the fluid enters is rigidly fixed to the frame. When the gauge is connected to the gauge point, fluid under pressure enters the tube.

→ Due to increase in internal pressure, the elliptical cross section of the tube tends to become circular. thus causing the tube to straighten out slightly.

→ The small outward movement of the free end of the tube is transmitted, through a link, quadrant and pinion, to a pointer.

→ The pointer moves clockwise on the graduated circular dial indicates the pressure intensity of the fluid.

→ The dial of the gauge is so calibrated that it reads zero when the pressure inside the tube equals to the local atmospheric pressure.

→

HYDROSTATICS (CHAPTER-3)

Total pressure :-

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces.

→ This force always acts normal to the surface.

centre of pressure :-

centre of pressure is defined as the point of application of the total pressure on the surface.

vertical plane surface submerged in liquid :-

consider a plane vertical surface submerged in a liquid.

A = total area of the surface

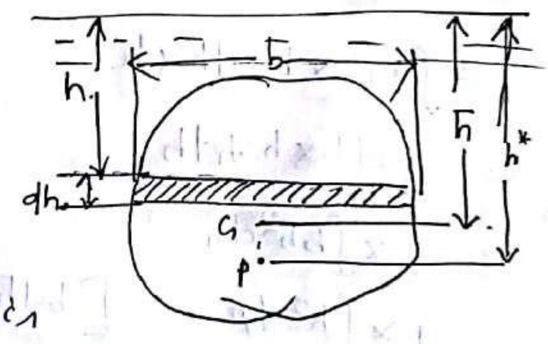
$\bar{h}$ = distance of C.G of the area from free surface of liquid.

G = centre of gravity of plane surface.

P = centre of pressure.

h^* = centre of pressure from free surface of liquid.

Total pressure :-



The total pressure on the surface may be determined by dividing the entire surfaces into a number of parallel strips.

→ The force on small strip is then calculated and the total pressure force on the whole area is calculated by integrating the force.

→ Consider a strip of thickness dh and width b at a depth of h from free surface of liquid.

pressure intensity on the strip $= \rho g h$

Area of the strip $= \boxed{dA = b \times dh}$

total force on the strip $dF = P \times area$

$= \rho g h \times b \times dh$

$F = \int dF = \int \rho g h \times b \times dh$

$= \int \rho g \times b h\, dh$

$= \rho g \int b h$

$= \rho g \times \int h \times dA$

$$F = \rho g \times A \times \bar{h}$$

A = Area of surface

$\bar{h}$ = distance of C.G from the free surface.

Centre of pressure h^*

→ Centre of pressure is calculated by using principle of moments.

→ Principle of moments state that the moment of the resultant force about an axis is equal to the sum of the moments of the components about the same axis.

→ The resultant force F is acting at 'P' at a distance h^* from free surface of the liquid.

→ Moment of the force 'F' about free surface $= F \times h^*$

→ Moment of force dF, acting on a strip about free surface $= dF \times h$.

$$= \rho g h \times b \times dh \times h.$$

Sum of forces of all such forces about free surface

$$= \int \rho g h \times b \times dh \times h.$$

$$= \rho g \int b \times h \times h dh$$

$$= \rho g \times \int b h^2 dh$$

$$= \rho g \times \int h^2 dA \quad [b dh = dA]$$

$$= \rho g \times I_0 \quad [I_0 = \int h^2 dA]$$

$$\therefore F \times h^* = \rho g \times I_0$$

$$\Rightarrow h^* = \frac{\rho g \times I_0}{F}$$

$$= \frac{\rho g \times I_0}{\rho g \times A \times \bar{h}} = \frac{I_0}{A\bar{h}}$$

$$h^* = \frac{I_0}{A\bar{h}}$$

from parallel axis theorem we have

$$I_0 = I_G + A \times \bar{h}^2$$

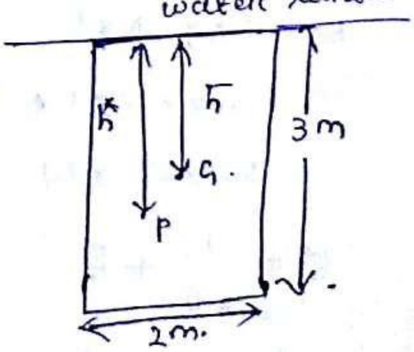
I_G = Moment of inertia of area about an axis passing through the C.G of the area and parallel to the free surface of the liquid.

$$h^* = \frac{I_G + A \times \bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

plane surface	C.G from the base	Area	I_G	I_0
1. Rectangle (b, d)	$x = d/2$	bd	$bd^3/12$	$bd^3/3$
2. Triangle (b, h)	$x = h/3$	$bh/2$	$bh^3/36$	$bh^3/12$
3. circle (d)	$x = d/2$	$\pi d^2/4$	$\pi d^4/64$	–

Q) A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and coincides with water surface.

(b) 2.5 m below the free water surface.



$F = \rho g \times A \times \bar{h}$

$A = b \times d$
$= 3 \times 2 = 6$

$\rho = 1000\ kg/m^3$

$g = 9.8\ m/s^2$

$A = 3 \times 2 = 6\ m^2$

$\bar{h} = (d/2) = 3/2 = 1.5$

$F = 1000 \times 9.81 \times 6 \times 1.5 = 88290\ N.$

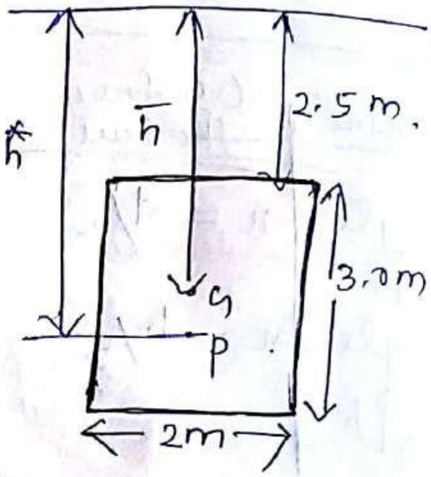
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{bh^3}{12} = \frac{2 \times 3^3}{12} = 4.5 m^4.$$

$$h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 2.0 m.$$

below water surface.

(b) upper edge is 2.5 m below water surface.



$F = \rho g \times A \times \bar{h}$

$\bar{h} = 2.5 + 3/2 = 4 m.$

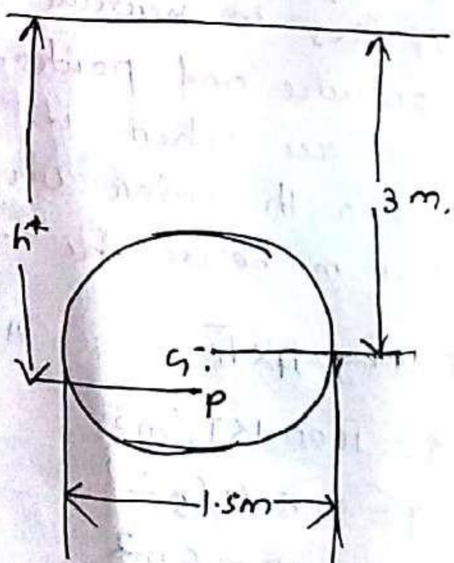
$F = 1000 \times 9.81 \times 6 \times 4$

$= 235440 N.$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$= \frac{4.5}{6 \times 4} + 4 = 4.1875 m.$$

Q) Determine the total pressure on a circular plate of diameter 1.5 m which is placed vertically in water in such a way that the centre of plate is 3 m below the free surface of water. find the position of centre of pressure.



$d = 1.5 m$

$A = \pi/4 \times (1.5)^2 = 1.767 m^2$

$\bar{h} = 3 m.$

$F = \rho \times g \times A \times \bar{h}$

$= 1000 \times 9.81 \times 1.767 \times 3$

$= 52002.8 N.$

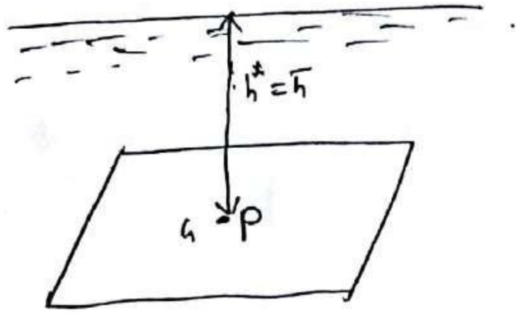
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$I_G = \pi/64 \, d^4 = 0.2485 m^4.$

$h^* = 3.0468 m.$

* Horizontal plane surface

→ Consider a plane horizontal surface immersed in a static fluid.



→ As 'G' and 'P' are at the same depth from the free surface of the liquid, ~~the pressure intensity~~

→ A = total area

Total force $F = \Delta p \times \text{Area}$

$= \rho g h \times A$

$$F = \rho g A \times \bar{h}$$

$$h = \bar{h} = h^*$$

Q

3.4 Archimede's principle :-

→ It states that when a body is immerged in a fluid either wholly or partially, it is lifted up by a force which is equal to the weight of the fluid displaced by the body.

→ According to Archimede's principle it is therefore known that the buoyant force is equal to the weight of the fluid displaced by the body.

BUOYANCY :-

→ when body is immerged in a fluid either wholly or partially it is subjected to an upward force which tends to lift it up. This tendency for an immerged body to be lifted up in the fluid due to an upward force opposite to the action of gravity is known as buoyancy.

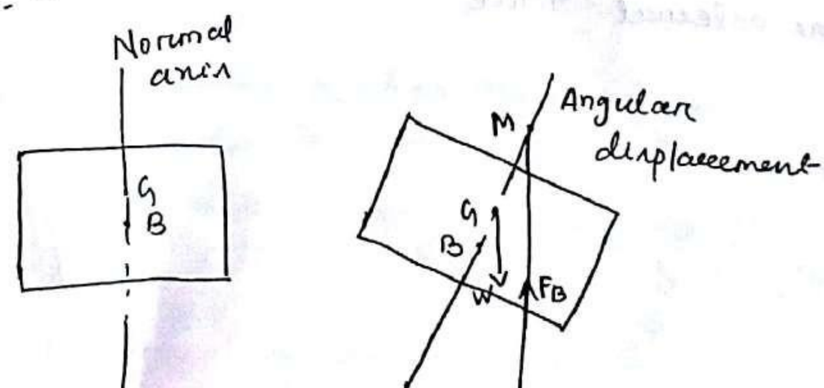
→ The force tending to lift up the body under such conditions is known as buoyant force.

Centre of bouoyancy :-

It is defined as the point through which the force of buoyancy is supposed to act.

→ The centre of buoyancy will be the centre of gravity of the fluid displaced.

Meta centre :-



→ It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.

→ The meta centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

→ consider a body floating in a liquid as shown in figure. Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy.

→ for equilibrium, both the points lie on the normal axis, which is vertical.

→

* The distance between the centre of gravity of a floating body and the metacentre (GM) is called metacentric height.

Types of equilibrium of floating bodies :—

the equilibrium of floating bodies is of following types.

① stable equilibrium

② unstable equilibrium

③ Neutral equilibrium.

Kinematics Of flow

→ Kinematics is defined as the branch of science which deals with motion of particles without considering the forces causing the motion.

→ The fluid motion is described by two methods.

① Lagrangian method.

② Eulerian method.

→ In the lagrangian method a single fluid particle is followed during its motion and its velocity, acceleration, density are described.

→ In Eulerian method the velocity, acceleration, pressure, density are described at a point. The Eulerian method is commonly used in fluid mechanics.

Types Of Flow :-

① steady and unsteady flow

② uniform and non-uniform flow.

③ Laminar and turbulent flow

④ Compressible and incompressible flow

⑤ rotational and irrotational flow

⑥ one, two and three dimensional flow.

① steady and unsteady flow :-

→ steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density at a point don't change with time.

→ for steady flow $\frac{\partial V}{\partial t} = 0$ $\frac{\partial P}{\partial t} = 0$, $\frac{\partial \gamma}{\partial t} = 0$.

→ unsteady flow is defined as that type of flow in which the velocity, pressure and density at a point changes with respect to time.

$\frac{\partial v}{\partial t} \neq 0$ $\frac{\partial \rho}{\partial t} \neq 0$.

② uniform and nonuniform flow:-

→ Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (length of direction of flow)

for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t=c} = 0$$

∂V = change of velocity

∂s = Length of flow in the direction

→ Non uniform flow is that type of flow in which the velocity at any given time changes with respect to space. for non uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t=c} \neq 0$$

③ compressible and incompressible flow:-

→ compressible flow is that type of flow in which density of fluid changes from point to point, the density (ρ) is not constant for the fluid.

$$\rho \neq C$$

→ Incompressible flow is that type of flow in which the density is constant for the fluid flow. for incompressible flow

$$\rho = C$$

④ Laminar and turbulent flow:-

→ Laminar flow is defined as that type of flow in which the fluid particles move along the stream line and all the stream lines are straight and parallel.

→ This type of flow are also called as streamline flow.

→ for ~~reynold no~~ laminar flow $\text{Raynold No} = \frac{VD}{\nu} < 2000$

→ Turbulent flow is that type of flow in which the fluid particles move in a zig zag way.

for turbulent flow $Re > 4000$

⑤ Rotational and irrotational flow :-

→ Rotational flow is that type of flow in which the fluid particles while flowing along the stream line also rotate about their own axis.

→ Irrotational flow is defined as that type of flow in which the fluid particles flowing along the stream line do not rotate about their own axis.

⑥ One-dimensional, two-dimensional, three-D flow :-

→ One-dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space coordinate only.

→ The variation of velocity in other two mutually ⊥r direction is assumed to be negligible.

→ $u = f(x)$, $v = 0$, and $w = 0$.

→ Two dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and two space coordinates such as x and y.

→ The variation of velocity in 3rd direction is negligible.

$$u = f_1(x, y), \quad v = f_2(x, y), \quad w = 0$$

→ 3 dimensional flow is that type of flow in which the velocity is a function of time and 3 ~~mutually~~ space coordinates.

$$u = f_1(x, y, z) \quad v = f_2(x, y, z) \quad w = f_3(x, y, z).$$

Rate of flow or Discharge :-

It is defined as the quantity of a fluid flowing per second through a section of a pipe.

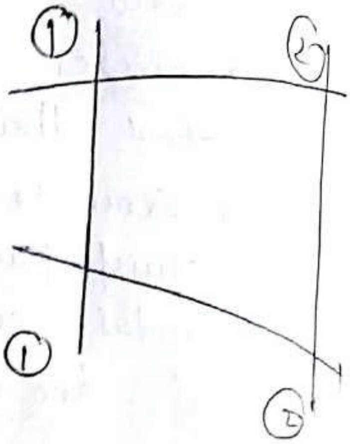
(m^3/s).

→ $\boxed{Q = A \times V}$

A = cross sectional area of pipe

V = average velocity of fluid.

Continuity Equation



→ The equation based on the principle of conservation of mass is called continuity equation.

→ Thus for a fluid flowing through the pipe at all cross-section, the quantity of fluid per second is constant.

→ consider 2 sections ① and ②

→ V_1 = average velocity at cross section 1-1

ρ_1 = density at section 1-1

A_1 = Area of pipe at 1-1

V_2 = average velocity at cross section 2-2

ρ_2 = density at section 2-2

A_2 = Area of pipe at 2-2

The rate of flow at section 1-1 = $\rho_1 A_1 V_1$

The rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to law of conservation of mass

rate of flow at section 1-1 = rate of flow at section 2-2

$$\boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

→ It is known as continuity equation.

if the fluid is incompressible

$$\boxed{\rho_1 = \rho_2}$$

$$\boxed{A_1 V_1 = A_2 V_2}$$

Q// The diameters of a pipe at the section 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section ① is 5m/s. Find velocity at section 2.

$Q = A_1 V_1 = 0.03927\ m^3/s$

$A_1 V_1 = A_2 V_2$

$\Rightarrow V_2 = 2.22\ m/s$

Q) A 30cm pipe containing water, branches into two pipes of diameters 20cm and 15cm respectively. If the avg. velocity in the 30cm pipe is 2.5m/s find the discharge in the pipe. Also determine the velocity in 15cm pipe if the avg. velocity in 20cm pipe is 2m/s.

$D_1 = 30cm = 0.3 m$

$A_1 = \pi/4 \, D_1^2 = 0.07068 \, m^2$

$V_1 = 2.5 m/s$.

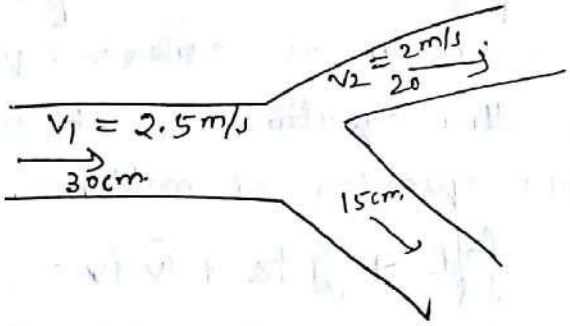
$D_2 = 20 cm = 0.2 m$.

$A_2 = \pi/4 \times (0.2)^2 = 0.0314 m^2$

$V_2 = 2 m/s$

$D_3 = 15 cm = 0.15 m$.

$A_3 = \pi/4 (0.15)^2 = \pi/4 \times 0.225 = 0.01767 m^2$.



$Q_1 = Q_2 + Q_3$

$Q_1 = A_1 V_1 = 0.1767 m^3/s$.

$Q_2 = A_2 V_2 = 0.0628 m^3/s$.

$Q_1 = Q_2 + Q_3$

$\Rightarrow Q_3 = 0.1139 m^3/s$.

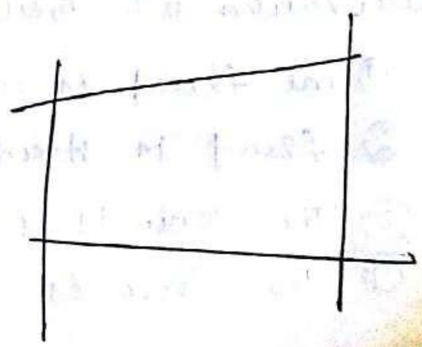
$Q_3 = A_3 \times V_3$

$\Rightarrow V_3 = 6.44 m/s$ (Ans).

Q) The diameters of a pipe at the sections ① and ② are 10cm and 15 cm respectively. Find the discharge through the pipe if the velocity

$d_1 = 10cm = 0.10 m$

$d_2 = 0.15 m$.



BERNOULLI'S EQUATION :

Euler's equation is derived by considering the gravity and pressure. and the motion of fluid element is considered along a stream line.

$$\boxed{\frac{dp}{\rho} + g\,dz + v\,dv = 0}$$

It is known as euler's equation of motion

Bernoulli's equation is obtained by integrating the Euler's equation of motion.

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = c$$

If flow is incompressible $\rho = c$

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = c$$

$$\Rightarrow \boxed{\frac{p}{\rho g} + z + \frac{v^2}{2g} = c}$$

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = Kinetic energy per unit weight or kinetic head.

z = potential head.

Assumptions :-

The following assumptions are taken on the derivation of Bernoulli's equation

1. The fluid is ideal
2. fluid is steady
3. The flow is incompressible
4. The flow is irrotational.

Q) Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 and with mean velocity of 2 m/s. Find the total head or total energy per unit weight of the water at a cross-section which is 5m above the datum line.

(Ans) Diameter of pipe = 5cm = 0.5m.

pressure $p = 29.43 N/cm^2 = 29.43 \times 10^4 N/m^2$.

velocity $V = 2.0 m/s$

Datum $z = 5 m$.

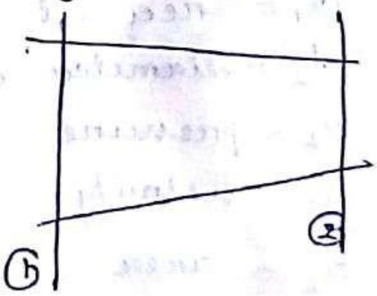
Total head = pressure head + Kinetic head + datum head

pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 m$.

velocity head $= \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 m$.

total head $= \frac{p}{\rho g} + \frac{V^2}{2g} + z = 30 + 0.204 + 5 = 35.204 m$. (Ans)

Q) A pipe through which water is flowing, is having diameters 20 cm and 10 cm at the cross sections ① and ② respectively. The velocity of water at section ① is given 4.0m/s. Find the velocity head at sections ① and ② and also rate of discharge.



$D_1 = 20 cm = 0.2m$

$A_1 = 0.0314 m^2$.

$V_1 = 4.0 m/s$.

$D_2 = 0.1 m/s$.

$A_2 = 0.00785 m^2$.

① velocity head at section ① $= \frac{V_1^2}{2g} = 0.815 m$.

② $A_1V_1 = A_2V_2$

$\Rightarrow V_2 = 16 m/s$

velocity head at section ② = 83.047 m.

discharge A_1V_1 or A_2V_2

= 0.1256 m^3/sec.

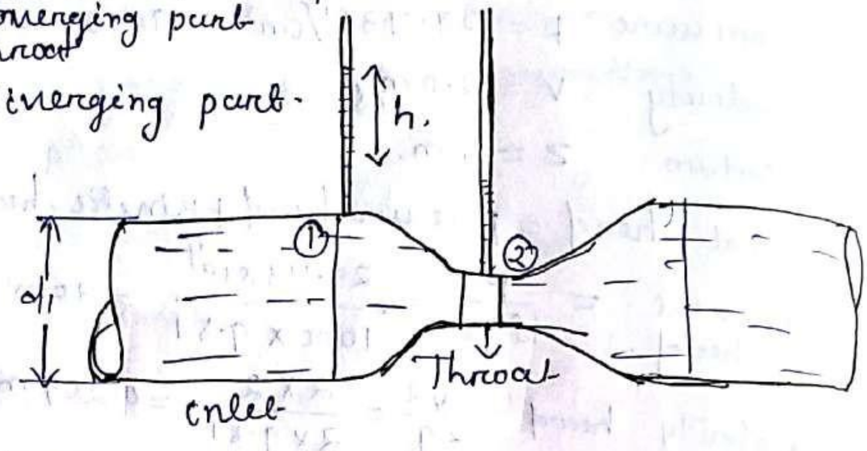
Practical Applications of Bernoulli's Theorem :-

① Venturimeter :-

→ A venturimeter is a device used for measuring the rate of flow flowing through a pipe.

It consists of three parts :

① Converging part
② Throat
③ Diverging part.



→ Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing.

d_1 = diameter at inlet ①
P_1 = pressure at section ①
V_1 = velocity of fluid at section ①
a_1 = area at section ①.
d_2 = diameter at section ②
P_2 = pressure ,, ,,
V_2 = velocity ,,
a_2 = area ,,

Applying Bernoulli's equation at ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

⇒ pipe is horizontal $(z_1 = z_2)$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}.$$

$$\Rightarrow \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$\therefore \frac{P_1 - P_2}{\rho g} = h$ (difference of pressure head).

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\Rightarrow v_2^2 - v_1^2 = 2gh$$

$$\Rightarrow \boxed{h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}}$$

Applying continuity equation

$$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \boxed{v_1 = \frac{a_2 v_2}{a_1}}$$

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g}$$

$$= \frac{v_2^2}{2g} - \frac{a_2^2 v_2^2}{a_1^2 \, 2g}$$

$$= \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right].$$

$$= \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$\Rightarrow \boxed{v_2 = \sqrt{2gh}\,\frac{a_1}{\sqrt{a_1^2 - a_2^2}}}$$

theoretical discharge

$$Q = a_2 \times v_2$$

$$\boxed{Q_{th} = \sqrt{2gh}\,\frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}}$$

$\because$ Actual discharge is less than theoretical discharge.

$$\boxed{Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}}$$

C_d = coefficient of discharge. $\boxed{C_d < 1}$

value of 'h' given by differential U-Tube manometer

$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

S_h = specific gravity of a heavy liquid.
S_o = specific gravity of liquid flowing through p.
x = difference of the heavier liquid column in U-tube.

If $S_h > S_o$

If $S_h < S_o$

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

S_l = sp. gravity of lighter liquid in U-tube.

Q) A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively. The reading of differential manometer connected to the inlet and the throat is 20cm of Hg. Determine the rate of flow

$C_d = 0.98$

$d_1 = 30\text{ cm}$ $a_1 = \pi/4\, d_1^2 = 706.85\text{ cm}^2$

$d_2 = 15\text{ cm}$ $a_2 = 176.7\text{ cm}^2$

$C_d = 0.98$.

$x = 20\text{ cm}$.

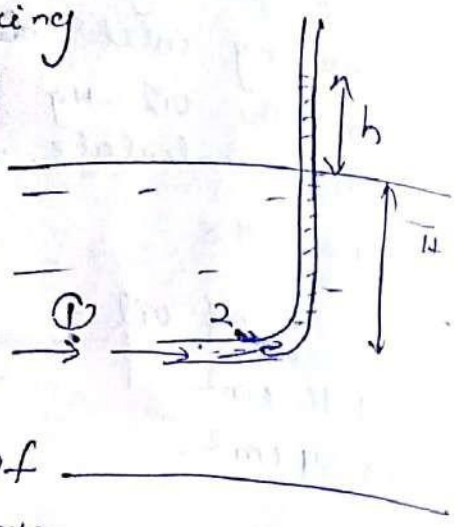
$$h = x\left[\frac{S_h}{S_o} - 1\right] = 20\left[\frac{13.6}{1} - 1\right] = 252.0\text{ cm. of } H_2O.$$

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$= 0.98 \times$ $= 125.756$ lt/sec.

pitot Tube

→ It is a device used for measuring the velocity of flow at any point in a pipe or a channel.



→ It is based on the pressure principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of kinetic energy into pressure energy.

→ pitot tube consists of a glass tube bent at right angle.

→ The lower end which is bent through 90° is directed in the upstream direction. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy.

→ The velocity is determined by measuring the rise of liquid in the tube.

→ consider the two points ① and ② at same level in such a way that the point ② is just as the inlet of the pitot tube and point ① is far away from the tube.

P_1 = intensity of pressure at point ①

V_1 = velocity of flow at ①

P_2 = pressure at point ②

V_2 = velocity at point ② = 0.

H = depth of tube in liquid

h = rise of liquid on the tube above the free surface.

Applying Bernoulli's equation at ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$\frac{P_1}{\rho g}$ = pressure head at ① = H

$\frac{P_2}{\rho g}$ = pressure head at ② = (H+h).

$z_1 = z_2$
$V_2 = 0$

$$H + \frac{V_1^2}{2g} = (h + H).$$

$$\Rightarrow \frac{V_1^2}{2g} = h$$

$$\Rightarrow V_1 = \sqrt{2gh}$$

$$(V_1)_{act} = C_V \times \sqrt{2gh}$$

C_V = coefficient of pitot tube.

— o —

Q/ A pitot tube is inserted in a pipe of 300mm diameter. The static pressure in pipe is 100mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, is 0.981 N/cm². Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take $C_V = 0.98$.

Ans $d = 300\,mm = 0.3\,m$.

$a = \pi/4\, d^2 = 0.07068\,m^2$

static pressure head = 100 mm of Hg (vacuum).

$$= \frac{-100}{1000} \times 13.6 = -1.36 \text{ m of water.}$$

$$\frac{-100}{1000} \times g \times 13.6 = (\rho)_w \times g \times h.$$

$$\Rightarrow h = \frac{\frac{-100}{1000} \times g \times 13.6 \times 10^3}{1000 \times g} = \frac{-100}{1000} \times 13.6 = -1.36 \text{ m of water.}$$

stagnation pressure head $= \frac{0.981 \times 10^4}{\rho g} = 1\,m.$

$h = (H+h) - H$
$= 1 - (-1.36) = 2.36$ m of water.

velocity at centre $= C_v \times \sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36}$

$= 6.668 \, m/s.$

mean velocity $= \bar{v} = 0.85 \times 6.668 = 5.6678 \, m/s.$

Rate of flow $= \bar{v} \times area = 0.4006 \, m^3/s$ (Ans)

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ORIFICE

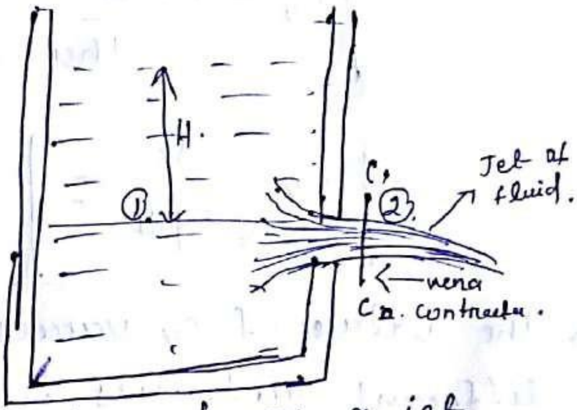
CHAPTER-5

→ orifice is a small opening of any cross section (such as circular, triangular, rectangular etc) on the side or at the bottom of a tank, through which the fluid is flowing.

→ ~~Classification of orifice~~ :-

Flow through an orifice :-

→ consider a ~~fluid~~ tank filled with a circular orifice in one of its sides.



→ Let H be the head of the liquid above the centre of the orifice.

→ The liquid flowing through the orifice forms a jet of liquid whose area of cross section is less than that of orifice.

→ The area of jet of fluid goes on decreasing and at a section C-C, the area is minimum. This section is approximately at a distance of half of diameter of the orifice.

→ At this section the stream lines are straight and parallel to each other. The section is called vena contracta.

→ Consider two points ① and ②. point ① is inside the tank and point ② is at venacontracta.

Applying Bernoulli's eq^n

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\boxed{Z_1 = Z_2}$$

$\frac{P_2}{\rho g} = 0$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}.$$

$V_1 << V_2$

$V_1 = 0$.

$$\Rightarrow H + 0 = 0 + \frac{V_2^2}{2g}$$

$$\Rightarrow H = \frac{V_2^2}{2g} \Rightarrow \boxed{V_2 = \sqrt{2gH}}$$ (theoretical velocity)

① coefficient of velocity (C_v) :- It is defined as the ratio between the actual velocity of jet of liquid at vena contracta and the theoretical velocity of jet.

$$C_v = \frac{\text{Actual velocity at vena contracta}}{\text{Theoretical velocity}}$$

$$C_v = \frac{V}{\sqrt{2gH}}$$

→ The value of C_v varies from 0.95 to 0.99 for different orifices.

Generally $C_v = 0.98$.

② coefficient of contraction (C_c)

It is defined as the ratio of the area of the jet at vena contracta to the area of the orifice. It is denoted by C_c.

$$C_c = \frac{\text{area of jet at vena contracta}}{\text{area of orifice}}$$

$$C_c = \frac{a_c}{a}$$

→ The value of C_c varies from 0.61 to 0.69.

In general $C_c = 0.64$

③ coefficient of discharge (C_d).

It is defined as the ratio of the actual discharge to the theoretical discharge. It is denoted by C_d.

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{theoretical velocity} \times \text{theoretical area}}$$

$$C_d = C_c \times C_v$$

⇒ C_d value varies from 0.61 to 0.65.

(0.62)

Q) The head of water over an orifice of diameter 40mm is 10m. find the actual discharge and the actual velocity of jet at vena contracta. $C_d = 0.6$, $C_v = 0.98$.

$H = 10cm$

$d = 40mm = 0.04m$ (dia of orifice)

$a = \frac{\pi}{4}d^2 = 0.0012560 m^2$

$C_d = \frac{Q_{act}}{Q_{th}} = 0.6$

$\Rightarrow Q_{th} = V_{th} \times$ (Area of orifice)

$V_{th} = \sqrt{2gH} = 14 m/s$

$Q_{th} = 14 \times 0.001256 = 0.01758 m^3/s$.

$Q_{act} = 0.6 \times 0.01758 = 0.01054 m^3/s$.

$C_v = \frac{V_{act}}{V_{th}} = 0.98$.

$\Rightarrow V_{act} = 0.98 \times 14 = 13.72 m/s$. (Ans)

Q) The head of water over the centre of an orifice of diameter 20mm is 1m. The actual discharge through the orifice is 0.85 lt/s. Find the C_d.

$a = 0.000314 m^2$

$H = 1m$

$Q = 0.85 lt/s = 0.00085 m^3/s$.

$V_{th} = 4.429 m/s$.

$Q_{th} = 4.429 \times 0.000314 = 0.00139 m^3/s$.

$C_d = \underline{0.61}$

NOTCHES AND WEIRS

Introduction :-

→ A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

→ It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

→ A weir is a concrete structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall with a sharp edge at the top.

→ The notch is of small size while the weir is of a bigger size.

→ The notch is generally made of metallic plate while the weir is made of concrete structure.

Classification :-

The notches are classified as

① According to the shape of ~~notch~~ opening

(i) rectangular notch
(ii) Triangular notch
(iii) Trapezoidal notch
(iv) stepped notch.

② According to the effect of the sides of nappe :-

(i) Notch with end contraction
(ii) Notch without end contraction.

Weirs are classified according to shape

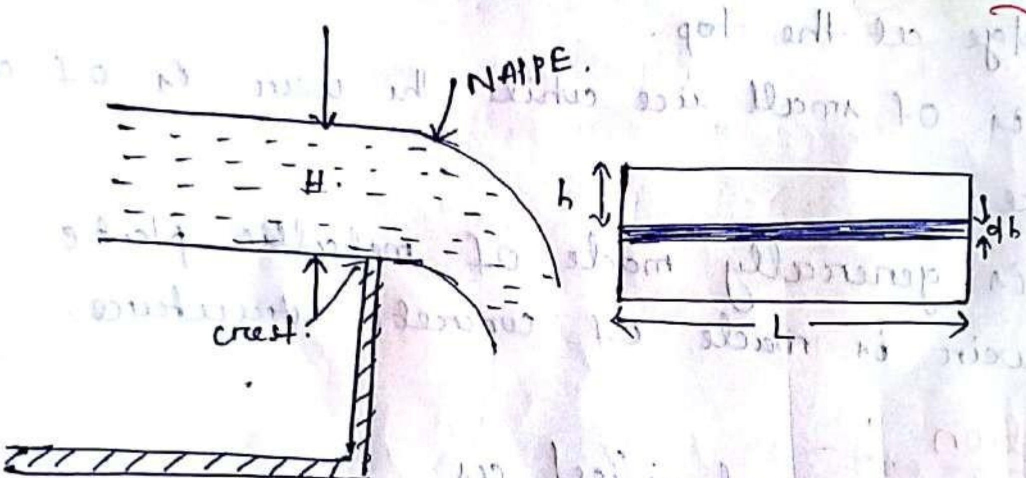
(a) According to the shape of opening

(i) Rectangular weir
(ii) Triangular weir
(iii) Trapezoidal weir.

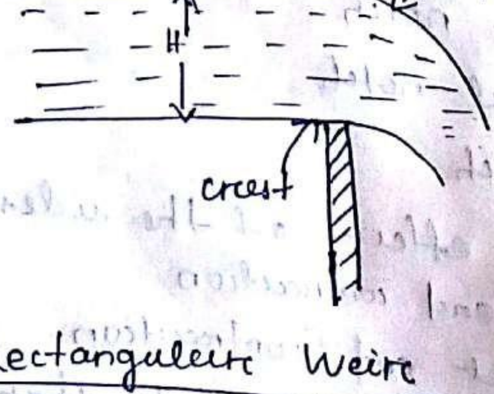
(b) According to the shape of crest
- (i) Sharp-crested weir
- (ii) Narrow-crested weir
- (iii) Board-crested weir
- (iv) Ogee-shaped weir.

(c) According to the effect of sides on the emerging nappe:-
- (i) weir with end contraction
- (ii) weir without end contraction.

DISCHARGE OVER A RECTANGULAR NOTCH OF WEIR



Rectangular Notch



Rectangular Weir

consider a rectangular notch or weir provided in a channel carrying water.

H = head of water over the crest

L = Length of the notch or weir.

→ To find the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface.

Area of strip $= L \times dh$.

theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$dQ = C_d \times$ area of strip $\times$ Theoretical velocity

$$\boxed{dQ = C_d \times L \times dh \times \sqrt{2gh}}$$

$$Q = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \times \int_0^H h^{1/2} dh$$

$$= C_d \times L \times \sqrt{2g} \times \left[\frac{h^{1/2+1}}{1/2+1}\right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \times \left[\frac{h^{3/2}}{3/2}\right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \times 2/3 \times (H)^{3/2}$$

$$Q = C_d \times L \times \sqrt{2g} \times 2/3 \times (H)^{3/2}$$

$$\boxed{Q = 2/3\, C_d L \sqrt{2g} \times (H)^{3/2}}$$

Q) Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300mm. $C_d = 0.60$.

Head over the notch $H = 300\,mm = 0.30\,m$

$L = 2\,m.$

$C_d = 0.60$

$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times (H^{3/2})$

$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times (0.30)^{3/2}$

$Q = 0.582\,m^3/s$

Q) Determine the height of a rectangular weir of length 6m to be built across a rectangular channel. The maximum length of water on the upstream side of the weir is 1.8 m and discharge is 2000 litre/s. Take $C_d = 0.6$.

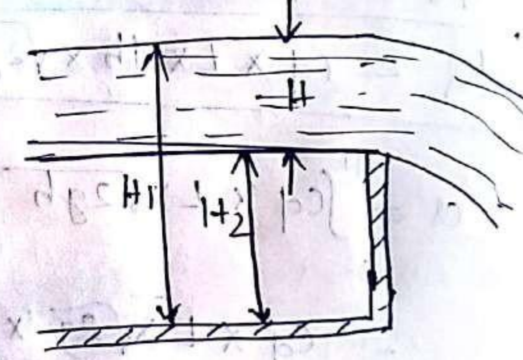
$L = 6\,m.$

$H_1 = 1.8\,m$

$Q = 2000\,lt/s.$

$= 2\,m^3/s.$

$C_d = 0.6$



$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \times H^{3/2}$

$\Rightarrow 2 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$

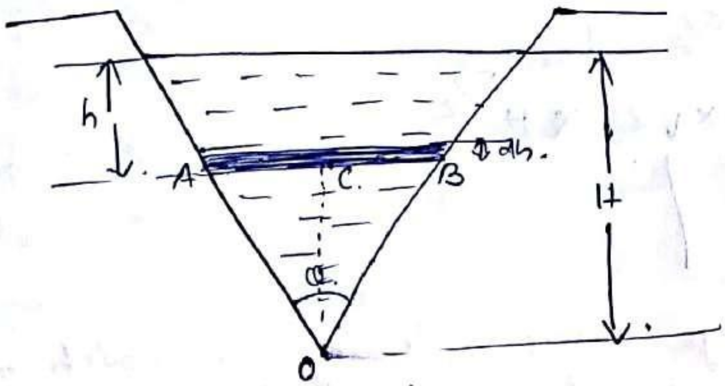
$\Rightarrow H^{3/2} = \frac{2.0}{10.623}$

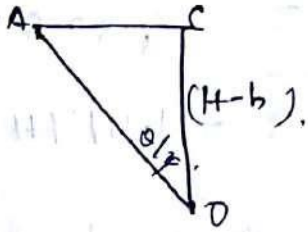
$H = 0.328\,m$

$H_2 = H_1 - H$

$= 1.8 - 0.328 = 1.472\,m.$ (Ans)

DISCHARGE OVER A TRIANGULAR NOTCH OR WEIR





H = head of water above the v-notch.

θ = angle of notch.

consider the horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water.

$$\tan \theta/2 = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$AC = (H-h) \tan \theta/2$$

$$AB = \text{width of strip} = 2 \times AC$$

$$= 2 \times (H-h) \tan \theta/2 \times dh$$

theoretical velocity of water through strip $= \sqrt{2gh}$

Discharge through the strip

$$dQ = C_d \times \text{Area of strip} \times \text{velocity}$$

$$= C_d \times 2(H-h) \tan \theta/2 \times dh \times \sqrt{2gh}$$

$$= 2 \times C_d \times (H-h) \tan \theta/2 \times \sqrt{2gh} \times dh$$

$$Q = \int_0^H 2 C_d \times (H-h) \tan \theta/2 \times \sqrt{2gh} \times dh$$

$$= 2 C_d \times \tan \theta/2 \times \sqrt{2g} \times \int_0^H (H-h)\, h^{1/2}\, dh$$

$$= 2 \times C_d \times \tan \theta/2 \times \sqrt{2g} \left[\int_0^H H h^{1/2} dh - \int_0^H h^{3/2} dh \right]$$

$$= 2 \times C_d \times \tan \theta/2 \times \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right].$$

$$= 2 \times C_d \times \tan \theta/2 \times \sqrt{2g} \times \left[\frac{4}{15} H^{5/2} \right].$$

$$Q = \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

for a V-notch $C_d = 0.6$

$\theta = 90^\circ$, $\tan\frac{\theta}{2} = 1$.

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2g} \times H^{5/2}$$

$$Q = 1.417 H^{5/2}$$

Q) Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. $C_d = 0.6$.

<u>Ans</u> $\theta = 60^\circ$

$H = 0.3\ m$,

$C_d = 0.6$

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan 30^\circ \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$Q = 0.040\ m^3/s$. (Ans)

FLOW THROUGH PIPES (CHAPTER-6)

Loss of energy in pipe: –

when a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as follows:

Energy loss

- Major energy loss
  - This is due to friction.
  - (a) Darcy-Weisbach formula.
  - (b) Chezy's Formula.
- Minor energy loss
  - (a) Sudden expansion of pipe
  - (b) Sudden contraction of pipe
  - (c) Bend in pipe.
  - (d) pipe fittings
  - (e) obstruction in pipe.

(I) Loss of energy due to friction.

(a) Darcy-Weisbach Formula: –

This loss of energy in pipes due to friction is calculated from Darcy-Weisbach equation.

$$h_f = \frac{4fLV^2}{2gd}$$

h_f = Loss of head due to friction.

f = Coefficient of friction $= \frac{16}{Re}$

$$f = \frac{16}{Re} \; (Re < 2000)$$

$$f = \frac{0.079}{Re^{1/4}} \; (Re \; (4000 - 10^6))$$

L = Length of pipe.

V = mean velocity of flow.

d = diameter of pipe.

(b) Chezy's formula

The expression for loss of head due to friction,

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

h_f = loss of head due to friction

A = area of cross-section of pipe

P = wetted perimeter of pipe

V = Mean velocity of flow.

P = perimeter of pipe.

L = Length of pipe.

$\frac{A}{P} = \frac{\text{Area of flow}}{\text{perimeter}}$ is called hydraulic mean depth

$\frac{A}{P}$ = (hydraulic mean depth or hydraulic radius)

(A/P) is denoted by 'm'.

$\therefore$ hydraulic mean depth $m = A/P = \frac{\pi/4\, d^2}{\pi d} = (d/4)$.

$\frac{A}{P} = m$ or $\left(\frac{P}{A}\right) = 1/m$.

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m}$$

$$\Rightarrow V^2 = h_f \times \left(\frac{\rho g}{f'}\right) \times m \times \frac{1}{L}$$

$$\Rightarrow V = \sqrt{\frac{\rho g}{f'} \times m \times \left(\frac{h_f}{L}\right)}$$

$$\boxed{V = \sqrt{\frac{\rho g}{f'}} \times \sqrt{m \frac{h_f}{L}}}$$

where $\sqrt{\frac{\rho g}{f'}} = C$ (C = chezy's constant)

$$\frac{h_f}{L} = i$$

$$\therefore \boxed{V = C \times \sqrt{m \times i}}$$

This is known as chezy's formula.

$\boxed{m = d/4}$

Q) Find the head lost due to friction in a pipe of diameter 300mm and length 50 m through which water is flowing at a velocity of 3m/s. using
(i) Darcy's formula
(ii) Chezy's formula. [Data $\nu = 0.01$ stoke]

$d = 300mm = 0.30m.$

$L = 50m.$ $\nu = 0.01$ stoke $= 0.01 \times 10^{-4} m^2/s$.

$v = 3m/s.$

$C = 60.$

$$Re = \frac{Vd}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5.$$

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256.$$

$$(i)\ h_f = \frac{4 \times f \times L \times v^2}{d \times 2g} \quad \text{(Darcy's formula)}$$

$$= \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2 \times 9.81}$$

$$\boxed{h_f = 0.7828\ m}\ (\text{Ans})$$

(ii) Chezy's formula.

$$V = C\sqrt{mi}$$

$$C = 60., \quad m = d/4 = \frac{0.3}{4} = 0.075\,m.$$

$$V = C \times \sqrt{mi}$$

$$\Rightarrow 3 = 60 \times \sqrt{0.075 \times \frac{h_f}{L}}$$

$$\Rightarrow \left(\frac{3}{60}\right)^2 = 0.075 \times \frac{h_f}{L}$$

$$\Rightarrow \frac{h_f}{L} = \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075}$$

$$\Rightarrow h_f = \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075} \times 50 = 1.665\ m.\ (\text{Ans})$$

Q) Find the diameter of pipe of length 2000 when the rate of flow of water through the pipe is 200 liters/sec and head loss due to friction is 4m.

$C = 50$

Ans $L = 2000\text{m}.$

$Q = 200 \text{ litre/s} = 0.2 \text{ m}^3/\text{s}.$

$h_f = 4\text{m}.$

$C = 50.$

$$V = \frac{\text{discharge}}{\text{Area}} = \frac{0.2}{(\frac{\pi}{4}d^2)}$$

$$V = C \times \sqrt{mi}$$

$$= 50 \times \sqrt{\frac{d}{4} \times \frac{h_f}{L}}.$$

$$\frac{0.2}{\frac{\pi}{4}d^2} = 50 \times \sqrt{\frac{d}{4} \times \frac{4}{2000}}$$

$$\Rightarrow \frac{0.2}{\frac{\pi}{4}d^2 \times 50} = \sqrt{\frac{d}{4} \times \frac{4}{2000}}$$

$$\Rightarrow \left(\frac{0.2 \times 4}{\pi d^2 \times 50}\right)^2 = \frac{d}{2000}$$

$$\Rightarrow \frac{(0.2)^2 \times (4)^2}{\pi^2 \times d^4 \times (50)^2} = \frac{d}{2000}$$

$$\Rightarrow \frac{(0.2)^2 \times 16 \times 2000}{\pi^2 \times (50)^2} = d^5$$

$$\Rightarrow d = \sqrt[5]{0.0518} = 0.553\text{m} = 553\text{mm} \text{ (Ans)}$$

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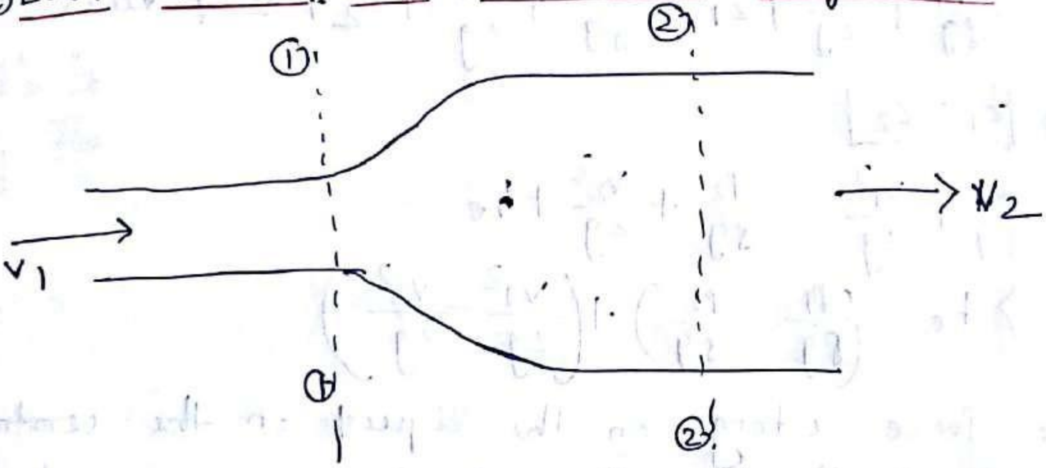
Minor Energy Losses :-

The loss of energy due to friction in pipe is known as major loss while the loss of energy due to change of velocity of the ~~fo~~ fluid is called minor loss of energy.

① Loss of head due to sudden enlargement



Consider a liquid flowing through a pipe which has sudden enlargement as shown in above figure. Consider two sections ①-① and ②-② before and after enlargement.

P_1 = pressure intensity at section ①-①

v_1 = velocity of flow at section ①-①

a_1 = area of pipe at section ①-①.

P_2 = pressure intensity at section ②-②

V_2 = velocity of flow at section ②-②

a_2 = area of pipe at section ②-②

→ Due to sudden change in diameter of pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the change of boundary. Thus the flow separates from the boundary and turbulent eddies are formed.

The loss of energy takes place due to forming of these eddies.

p' = pressure intensity of the liquid eddies

h_e = Loss of head due to sudden enlargement

Applying Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{head loss}$$

$\Rightarrow$ $\boxed{z_1 = z_2}$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$\Rightarrow h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

→ The force acting on the liquid in the control volume in the direction of flow is given by

$$\boxed{F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2}$$

→ $\underline{p' = p_1}$

$$F_x = p_1 A_1 + p_1 (A_2 - A_1) - p_2 A_2 = p_1 A_2 - p_2 A_2$$

$$\boxed{F_x = A_2 (p_1 - p_2)}$$

Momentum of liquid in section 1-1 $= \rho_1 A_1 V_1^2$

Momentum of liquid at section 2-2 $= \rho A_2 V_2^2$

change in momentum $= \rho A_2 V_2^2 - \rho A_1 V_1^2$

continuity equation $\boxed{A_1 V_1 = A_2 V_2}$

$$\boxed{A_1 = \frac{A_2 V_2}{V_1}}$$

Change in momentum/sec $= \rho A_2 V_2^2 - \rho \times A_1 V_1^2$

$$= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_2^2$$

$$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 (V_2^2 - V_1 V_2)$$

Net force acting on control volume in the direction of flow must be equal to the rate of change of momentum :

$$(p_1 - p_2) A_2 = \rho A_2 (V_2^2 - V_1 V_2)$$

$$\Rightarrow \frac{p_2 - p_2}{\rho} = V_2^2 - V_1 V_2$$

$$\Rightarrow \boxed{\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}}$$

$$\therefore h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

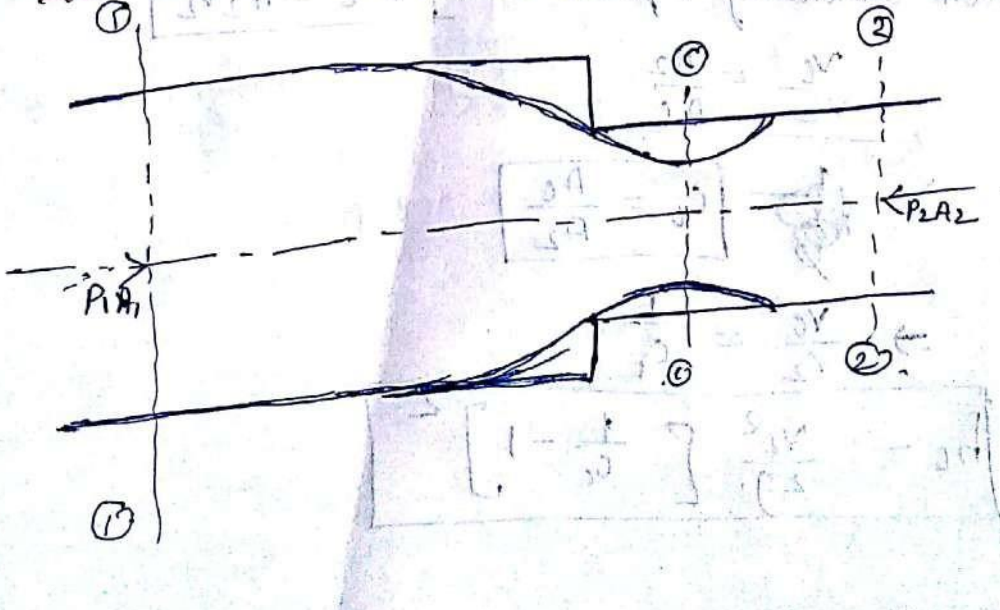
$$= \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$= \frac{2V_2^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 + V_1^2 - 2V_1 V_2}{2g}$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

Loss of Head due to sudden Contraction :-



→ Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in fig.

→ consider two section (1-1) and (2-2) before and after contraction.

→ As the liquid goes from a large pipe to a small pipe, the area of flow goes on decreasing and becomes minimum at section (c-c). This section is called as vena contracta.

→ After section c-c, a sudden enlargement takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena contracta to smaller pipe.

Let A_c = Area of flow at section c-c.

V_c = velocity of flow at section c-c.

A_2 = Area of flow at section 2-2.

V_2 = velocity of flow at section 2-2.

h_c = Loss of head due to sudden contraction.

$$h_c = \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

from continuity equation $\boxed{A_c V_c = A_2 V_2}$

$$\frac{V_c}{V_2} = \frac{A_2}{A_c}$$

$$\Rightarrow \boxed{C_c = \frac{A_c}{A_2}}$$

$$\Rightarrow \frac{V_c}{V_2} = \frac{1}{C_c}$$

$$\boxed{h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

where $K = \left[\frac{1}{C_c} - 1\right]^2$

$$h_c = \frac{K V_2^2}{2g}$$

$C_c = 0.62$

$\therefore K = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$

$$h_c = 0.375 \frac{V_2^2}{2g}$$

If the C_c value is not given then

$$h_c = 0.5 \frac{V_2^2}{2g}$$

Q) Find the loss of head when the pipe of diameter 200mm is suddenly enlarged to a diameter of 400mm. The rate of flow of water through the pipe is 250 litre/sec.

$D_1 = 200mm = 0.2m$

$D_2 = 400mm = 0.4m$

$A_1 = \pi/4\, D_1^2 = \pi/4 \times (0.2)^2 = 0.03141 m^2$

$A_2 = \pi/4\, D_2^2 = \pi/4 \times (0.4)^2 = 0.12564 m^2$.

$Q = 250 litre/s = 0.25 m^3/s$.

$V_1 = Q/A_1 = 7.96 m/s$.

$V_2 = Q/A_2 = 1.99 m/s$.

$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81}$

$= 1.816$ m of water. (Ans)

3) Loss of Head at the Entrance of pipe.

This is the loss of energy which occurs when a liquid enters a pipe which is connected to large tank.

$$h_i = 0.5 \frac{v^2}{2g}$$

v = velocity of liquid in pipe.

4) Loss of Head at the Exit of pipe :-

This is loss of head due to velocity of liquid at the outlet of pipe. It is denoted as h_o

$$h_o = \frac{v^2}{2g}$$

v = velocity of liquid at outlet of pipe.

5) Loss of head due to Bend in pipe :-

When there is bend in pipe, the velocity of flow changes due to which formation of eddies takes place.

$$h_b = \frac{Kv^2}{2g}$$

h_b = loss of head due to bend.
v = velocity of flow.
K = coefficient of bend.

6) Loss of Head in various pipe fittings :-

This is the loss of head in various pipe fittings. It is expressed as

$$\frac{Kv^2}{2g}$$

V = velocity of flow.
K = coefficient of pipe fitt.

HYDRAULIC GRADIENT LINE :-

It is defined as the line which gives the sum of pressure head (P/w) and datum head (Z) of a flowing fluid in a pipe with respect to some reference line.

→ It is briefly written as H.G.L (Hydraulic gradient Line)

TOTAL ENERGY LINE :-

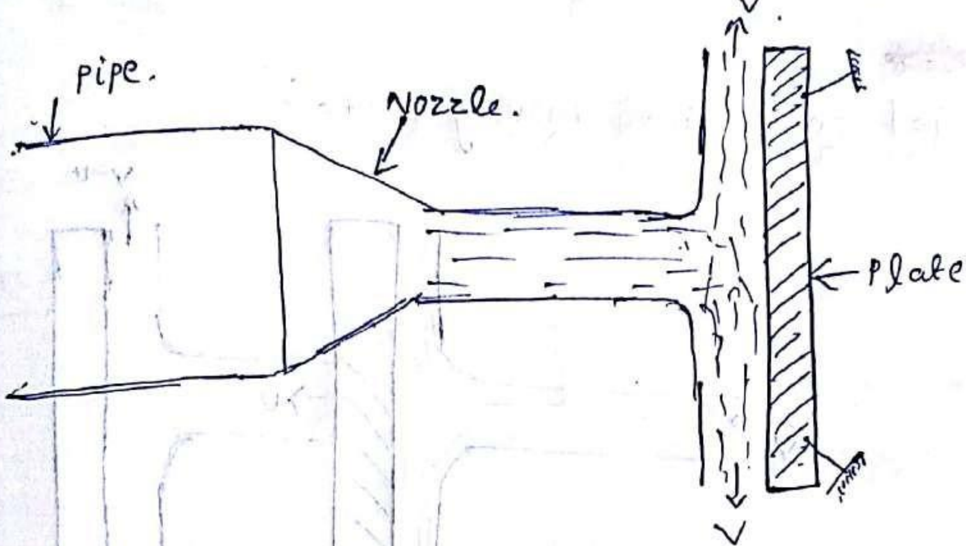
It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

→ It is briefly written as T.E.L (Total Energy Line).

— o —

IMPACT OF JET (CHAPTER-7)

Impact of jet on a fixed vertical flat plate



→ consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

V = velocity of the jet.

d = diameter of the jet

a = area of crossection of jet $= \frac{\pi}{4} d^2$

→ The jet after striking the plate, will move along the plate. But the plate is at right angles to the jet. Hence the jet after striking will get deflected through $90°$.

→ After striking the component of the velocity of jet in the direction of jet is zero.

The force exerted by the jet on the plate in the direction of jet

F_x = Rate of change of momentum in direction of force

$$= \frac{\text{Initial momentum} - \text{final momentum}}{\text{Time}}$$

$$= \frac{(\text{Mass} \times \text{initial velocity}) - (\text{Mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} (\text{initial velocity} - \text{Final velocity})$$

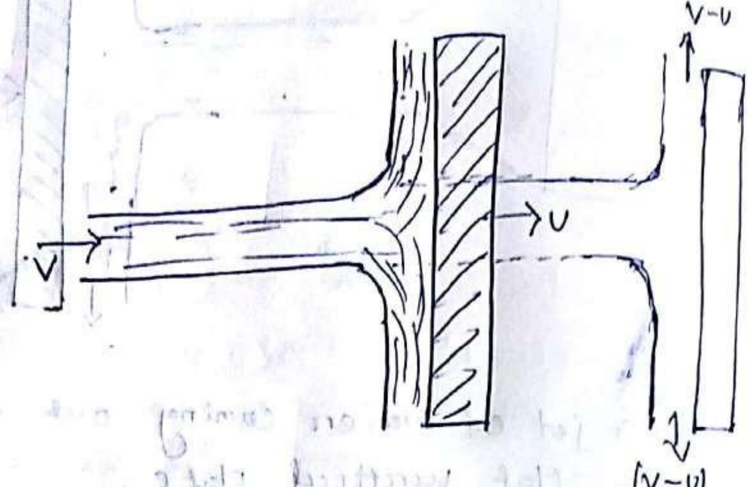
$$= \frac{\text{Mass}}{\text{time}} (V - 0)$$

$F_x = \rho a v (v - 0)$. $\left[mass/sec = \rho \times av \right]$

$$\boxed{F_x = \rho a v^2}$$

~~Force on Plate~~

Impact of jet on vertical moving plate :-



→ V = velocity of jet

a = area of cross-section of the jet

U = velocity of flat plate.

→ In this case, the jet does not strike the plate with a velocity V, but it strikes with a relative velocity.

→ The relative velocity is equal to the difference of absolute velocity of jet of water and the velocity of plate.

→ The relative velocity = $(V - U)$

→ Mass of water striking the plate per sec =

$\rho \times$ Area of jet $\times$ velocity with which jet strikes the plate.

$= \rho a (V - U)$.

→ Force exerted by the jet on the moving plate in the direction of jet

F_x = Mass of water × (initial velocity − final velocity)

$= \rho a (V - U) \left[(V - U) - 0 \right]$.

$$\boxed{F_x = \rho a (V - U)^2}$$

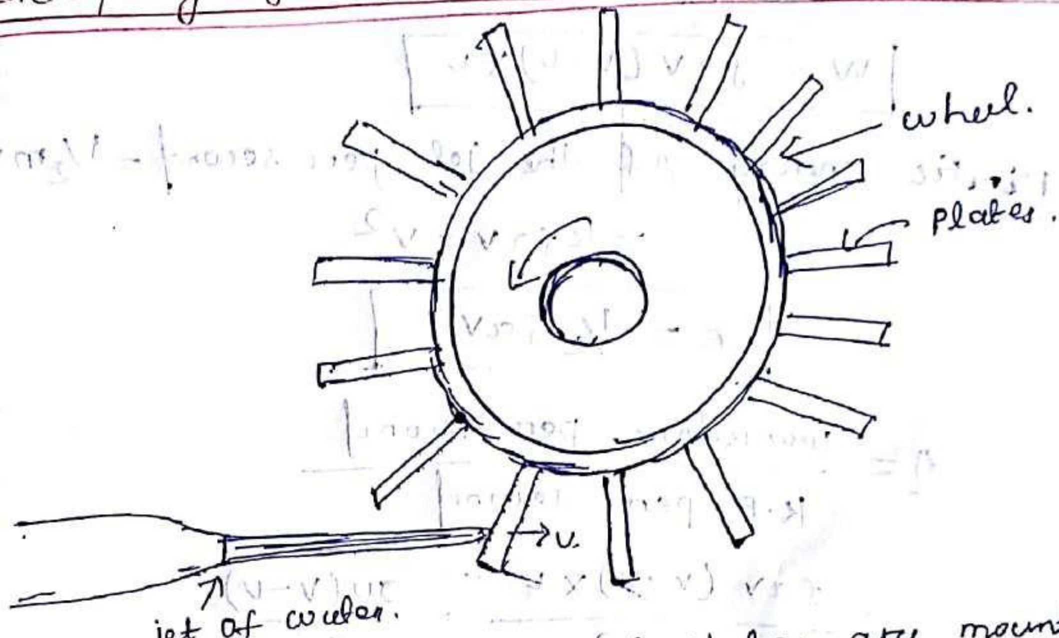
→ The work will be done by the jet on the plate, as plate is moving.

workdone = Force x velocity

$= F_x \times U$

$$W = \rho a(V-U)^2 \times U$$

Force exerted by a jet of water on a series of vanes



→ In actual practice, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart.

→ The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving.

V = velocity of jet.

d = diameter of jet.

a = cross-sectional area of jet = $\pi/4 d^2$

u = velocity of vane.

→ Mass of water per second striking the series of plates = $\rho a V$.

→ jet strikes the plate with a velocity = $(V-U)$

→ The force exerted by the jet in the direction of motion of plate

F_x = Mass ~~of~~ × (initial velocity − final velocity)

$= \rho a v[(V-u)-0]$

$$\boxed{F_x = \rho a v(V-u)}$$

workdone = Force × ~~Distance~~ velocity

$= F_x \times u$

$$\boxed{W = \rho a v(V-u) \times u}$$

Kinetic energy of the jet per second $= \frac{1}{2} m v^2$

$= \frac{1}{2} \rho a v \times v^2$

$$\boxed{KE = \frac{1}{2} \rho a v^3}$$

$$\eta = \frac{\text{workdone per second}}{\text{K.E per second}}$$

$$= \frac{\rho a v (v-u) \times u}{\frac{1}{2} \rho a v^3} = \frac{2u(v-u)}{v^2}$$

$$\boxed{\eta = \frac{2u(v-u)}{v^2}}$$

condition for Maximum Efficiency:-

$$\frac{d\eta}{du} = 0$$

$$\Rightarrow \frac{d}{du}\left[\left(\frac{2u(v-u)}{v^2}\right)\right] = 0$$

$$\Rightarrow \frac{d}{du}\left(\frac{2uv - 2u^2}{v^2}\right) = 0$$

$$\Rightarrow \frac{2v - 2 \times 2u}{v^2} = 0$$

$$\Rightarrow 2v - 4u = 0$$

$$\Rightarrow v = 2u \Rightarrow \boxed{u = v/2}$$

Maximum efficiency

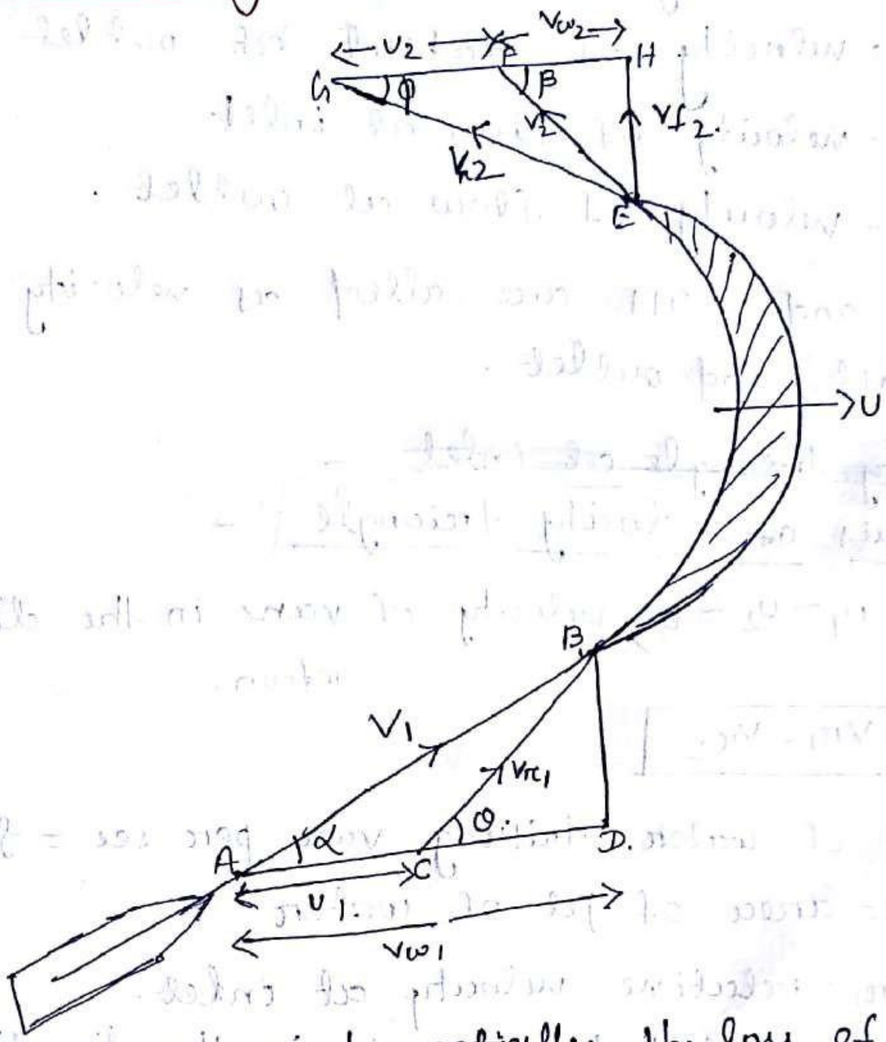
$$\eta_{max} = \frac{2v(V-v)}{V^2}$$

$$= \frac{2v(2v-v)}{(2v)^2}$$

$$= \frac{2v \times v}{4v^2} = 1/2 = 50\%$$

$$\boxed{\eta_{max} = 50\%}$$

Impact on a moving curved plate :-



→ As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. as the plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate.

V_1 = velocity of the jet at inlet

u_1 = velocity of vane at inlet

Vr_1 = relative velocity of jet and plate at inlet.

α = blade angle (inlet)

θ = vane angle (inlet).

V_2 = velocity of jet at outlet.

u_2 = velocity of vane at outlet.

Vr_2 = relative velocity of jet at outlet.

β = blade angle at outlet.

ϕ = vane angle at outlet.

V_{w_1} = velocity of whirl at inlet

V_{w_2} = velocity of whirl at outlet

V_{f_1} = velocity of flow at inlet

V_{f_2} = velocity of flow at outlet.

ABD and EGH are called as velocity triangles at inlet and outlet.

~~velocity triangle at inlet~~ :-

Analysis of velocity triangle :-

$u_1 = u_2 = u$ = velocity of vane in the direction of motion.

$$\boxed{V_{r_1} = V_{r_2}}$$

→ Mass of water striking vane per sec = $\rho a V_{r_1}$

a = area of jet of water.

V_{r_1} = relative velocity at inlet.

↳ force exerted by the jet in the direction of moti

$$\boxed{F_x = \rho a V_{r_1} (V_{w_1} + V_{w_2})}$$

If $\beta = 90^\circ$, $V_{w_2} = 0$

$$\boxed{F_x = \rho a V_{r_1} \times V_{w_1}}$$

→ β is obtuse angle

$$\boxed{F_x = \rho a V_{r_1} [V_{w_1} - V_{w_2}]}$$

Thus in general F_x can be written as

$$\boxed{F_x = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}]}$$

→ workdone per second on the vane by jet

$$= F_x \times U$$

$$\boxed{W = \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times U}$$

efficiency of jet :-

$$\eta = \frac{\text{output}}{\text{Input}}$$

$$= \frac{\text{workdone per second on the vane}}{K \cdot E}$$

$$= \frac{\rho a V_{r_1} (V_{w_1} \pm V_{w_2}) \times U}{\frac{1}{2} m V_1^2}$$

$$= \frac{\rho \cdot a V_{r_1} (V_{w_1} \pm V_{w_2}) \times U}{\frac{1}{2} \times \rho a V_{r_1} \times V_1^2}$$

$$= \frac{(V_{w_1} \pm V_{w_2}) \times U}{\frac{1}{2} \times V_1^2}$$

$$\boxed{\eta = \frac{2U(V_{w_1} \pm V_{w_2})}{V_1^2}}$$